12 Afgeleide en tweede afgeleide

12.1 De afgeleide van gebroken functies

bladzijde 108

1 **a** $x = 1$ maakt de noemer nul dus $f(1)$ bestaat niet.
x vlakbij 1 maakt de functiewaarde heel groot of heel klein, dus $x = 1$ is verticale asymptoot.
b Waarschijnlijk niet.
c Voer in $y_1 = \dfrac{x^2 - 2x + 5}{x - 1}$.
De optie maximum geeft $x = -1$ en $y = -4$.
De optie minimum geeft $x = 3$ en $y = 4$.
Dus de toppen zijn $(-1, -4)$ en $(3, 4)$.

bladzijde 109

2 **a** $f'(0) = -3$.
$\left.\begin{array}{l}\text{raaklijn: } y = -3x + b \\ f(0) = -5\end{array}\right\}$ raaklijn: $y = -3x - 5$
b $f'(x) = \frac{3}{4}$
$\dfrac{x^2 - 2x - 3}{(x-1)^2} = \frac{3}{4}$
$4x^2 - 8x - 12 = 3(x^2 - 2x + 1)$
$4x^2 - 8x - 12 = 3x^2 - 6x + 3$
$x^2 - 2x - 15 = 0$
$(x + 3)(x - 5) = 0$
$x = -3 \vee x = 5$
Dit geeft de punten $(-3, -5)$ en $(5, 5)$.

3 **a** $f(x) = \dfrac{x-2}{x+5}$ geeft $f'(x) = \dfrac{(x+5)\cdot 1 - (x-2)\cdot 1}{(x+5)^2} = \dfrac{x+5-x+2}{(x+5)^2} = \dfrac{7}{(x+5)^2}$

b $g(x) = \dfrac{3x+2}{-x+6}$ geeft $g'(x) = \dfrac{(-x+6)\cdot 3 - (3x+2)\cdot -1}{(-x+6)^2} = \dfrac{-3x+18+3x+2}{(-x+6)^2} = \dfrac{20}{(-x+6)^2}$

c $h(x) = \dfrac{2}{2x-1}$ geeft $h'(x) = \dfrac{(2x-1)\cdot 0 - 2\cdot 2}{(2x-1)^2} = \dfrac{-4}{(2x-1)^2}$

d $j(x) = \dfrac{6x-9}{3} = 2x - 3$ geeft $j'(x) = 2$

4 **a** $f(x) = \dfrac{x^3}{2x^2+1}$ geeft $f'(x) = \dfrac{(2x^2+1)\cdot 3x^2 - x^3\cdot 4x}{(2x^2+1)^2} = \dfrac{6x^4+3x^2-4x^4}{(2x^2+1)^2} = \dfrac{2x^4+3x^2}{(2x^2+1)^2}$

b $g(x) = \dfrac{x-2}{3-x^2}$ geeft $g'(x) = \dfrac{(3-x^2)\cdot 1 - (x-2)\cdot -2x}{(3-x^2)^2} = \dfrac{3-x^2+2x^2-4x}{(3-x^2)^2} = \dfrac{x^2-4x+3}{(3-x^2)^2}$

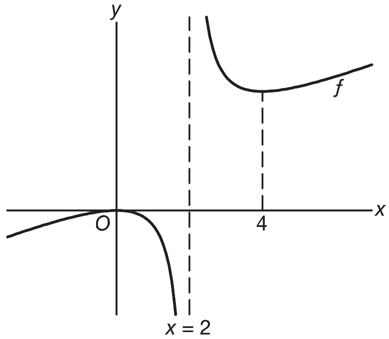
c $h(x) = \dfrac{3-x^2}{x-2} + x^3$ geeft $h'(x) = \dfrac{(x-2)\cdot -2x - (3-x^2)\cdot 1}{(x-2)^2} + 3x^2 = \dfrac{-2x^2+4x-3+x^2}{(x-2)^2} + 3x^2 = \dfrac{-x^2+4x-3}{(x-2)^2} + 3x^2$

d $j(x) = x - \dfrac{2}{x+4}$ geeft $j'(x) = 1 - \dfrac{(x+4)\cdot 0 - 2\cdot 1}{(x+4)^2} = 1 - \dfrac{-2}{(x+4)^2} = 1 + \dfrac{2}{(x+4)^2}$

5 **a** $f(x) = \dfrac{x^2}{x-2}$ geeft $f'(x) = \dfrac{(x-2)\cdot 2x - x^2\cdot 1}{(x-2)^2} = \dfrac{2x^2-4x-x^2}{(x-2)^2} = \dfrac{x^2-4x}{(x-2)^2}$

$f'(x) = 0$ geeft $\dfrac{x^2-4x}{(x-2)^2} = 0$

$x^2 - 4x = 0$
$x(x-4) = 0$
$x = 0 \vee x = 4$



max. is $f(0) = 0$
min. is $f(4) = 8$

b $\dfrac{x^2}{x-2} = p$ heeft geen oplossingen voor $0 < p < 8$.

c $f'(6) = \dfrac{36-24}{(6-2)^2} = \frac{12}{16} = \frac{3}{4}$

$\left.\begin{array}{l} \text{raaklijn: } y = \frac{3}{4}x + b \\ f(6) = \frac{36}{4} = 9 \text{ dus } A(6, 9) \end{array}\right\}\ 9 = \frac{3}{4}\cdot 6 + b$

$9 = 4\frac{1}{2} + b$

$4\frac{1}{2} = b$

Dus raaklijn: $y = \frac{3}{4}x + 4\frac{1}{2}$.

d $f'(x) = -3$ geeft $\dfrac{x^2-4x}{(x-2)^2} = -3$

$x^2 - 4x = -3(x^2 - 4x + 4)$
$x^2 - 4x = -3x^2 + 12x - 12$
$4x^2 - 16x + 12 = 0$
$x^2 - 4x + 3 = 0$
$(x-1)(x-3) = 0$
$x = 1 \vee x = 3$

Dit geeft de punten $(1, -1)$ en $(3, 9)$.

6 **a** $f(x) = 0$ geeft $\dfrac{x^2-4}{2x+5} = 0$

$x^2 - 4 = 0$
$x^2 = 4$
$x = 2 \vee x = -2$

Dus $A(-2, 0)$.

$f(x) = \dfrac{x^2-4}{2x+5}$ geeft $f'(x) = \dfrac{(2x+5)\cdot 2x - (x^2-4)\cdot 2}{(2x+5)^2} = \dfrac{4x^2+10x-2x^2+8}{(2x+5)^2} = \dfrac{2x^2+10x+8}{(2x+5)^2}$

Stel k: $y = ax + b$ met $a = f'(-2) = \dfrac{8-20+8}{1^2} = -4$.

$\left.\begin{array}{l} k\text{: } y = -4x + b \\ A(-2, 0) \end{array}\right\}\ 0 = -4\cdot -2 + b$

$-8 = b$

Dus k: $y = -4x - 8$.

b $\left.\begin{array}{l} f(0) = -\frac{4}{5} \text{ dus } B(0, -\frac{4}{5}) \\ f'(0) = \frac{8}{25} \end{array}\right\}\ l\text{: } y = \frac{8}{25}x - \frac{4}{5}$

c $f'(x) = 0$

$\dfrac{2x^2+10x+8}{(2x+5)^2} = 0$

$2x^2 + 10x + 8 = 0$
$x^2 + 5x + 4 = 0$
$(x + 1)(x + 4) = 0$
$x = -1 \ \lor \ x = -4$

$f(-1) = \dfrac{1-4}{-2+5} = -1$

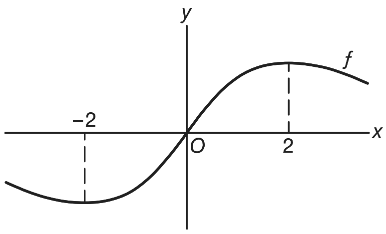
$f(-4) = \dfrac{16-4}{-8+5} = -4$

De punten met een horizontale raaklijn zijn $(-1, -1)$ en $(-4, -4)$.

bladzijde 110

7 **a** $f(x) = \dfrac{5x}{x^2+4}$ geeft $f'(x) = \dfrac{(x^2+4)\cdot 5 - 5x\cdot 2x}{(x^2+4)^2} = \dfrac{5x^2+20-10x^2}{(x^2+4)^2} = \dfrac{-5x^2+20}{(x^2+4)^2}$

$f'(x) = 0$ geeft $-5x^2 + 20 = 0$
$x^2 = 4$
$x = -2 \ \lor \ x = 2$



min. is $f(-2) = -1\frac{1}{4}$

max. is $f(2) = 1\frac{1}{4}$

b $f'(x) = \frac{3}{5}$ geeft $\dfrac{-5x^2+20}{(x^2+4)^2} = \frac{3}{5}$

$3(x^4 + 8x^2 + 16) = -25x^2 + 100$
$3x^4 + 24x^2 + 48 = -25x^2 + 100$
$3x^4 + 49x^2 - 52 = 0$
Stel $x^2 = p$.
$3p^2 + 49p - 52 = 0$
$D = 49^2 - 4 \cdot 3 \cdot -52 = 3025$, dus $\sqrt{D} = 55$

$p = \dfrac{-49-55}{6} \ \lor \ p = \dfrac{-49+55}{6}$

$p = -17\frac{1}{3} \qquad \lor \ p = 1$

$x^2 = -17\frac{1}{3} \qquad \lor \ x^2 = 1$

geen opl. $\qquad x = 1 \ \lor \ x = -1$

De x-coördinaten zijn 1 en -1.

8 **a** De verticale asymptoot is $x = 100$, dus 100% verwijderen is niet mogelijk.

b $K = \dfrac{0{,}6x}{100-x}$ geeft $\dfrac{dK}{dx} = \dfrac{(100-x)\cdot 0{,}6 - 0{,}6x\cdot -1}{(100-x)^2} = \dfrac{60-0{,}6x+0{,}6x}{(100-x)^2} = \dfrac{60}{(100-x)^2}$

c Van $\dfrac{dK}{dx} = \dfrac{60}{(100-x)^2}$ zijn teller en noemer positief en dus ook de breuk voor elke x tussen 0 en 100, dus is K stijgend voor elke x.

bladzijde 111

9 **a** $T = \dfrac{1200t + 10000}{t^2 + 1000}$ geeft

$$\frac{dT}{dt} = \frac{(t^2+1000)\cdot 1200 - (1200t + 10000)\cdot 2t}{(t^2+1000)^2}$$

$$= \frac{1200t^2 + 1200000 - 2400t^2 - 20000t}{(t^2+1000)^2} = \frac{-1200t^2 - 20000t + 1200000}{(t^2+1000)^2}$$

$\left[\dfrac{dT}{dt}\right]_{t=20} = \dfrac{320000}{1400^2} > 0$, dus T is stijgend op $t = 20$.

b $\left[\dfrac{dT}{dt}\right]_{t=30} = \dfrac{-480000}{1900^2} \approx -0{,}13296$ °C/minuut

Dus de afnamesnelheid is $0{,}13296 \times 60 \approx 8{,}0$ °C/uur.

10 **a** $P(4{,}5) \approx 140{,}91$

$P(6{,}5) \approx 143{,}33$

De procentuele toename is $\dfrac{P(6{,}5) - P(4{,}5)}{P(4{,}5)} \times 100\% \approx 1{,}7\%$.

b $\dfrac{dP}{dx} = -\dfrac{(1+x)\cdot 0 - 50\cdot 1}{(1+x)^2} = \dfrac{50}{(1+x)^2}$

Van $\dfrac{dP}{dx}$ zijn teller en noemer positief en dus ook de breuk voor elke x met $0 \le x \le 10$, dus de grafiek van P is stijgend.

c $\dfrac{dP}{dx} = 0{,}8$ geeft $\dfrac{50}{(1+x)^2} = \dfrac{4}{5}$

$4(1+x)^2 = 250$

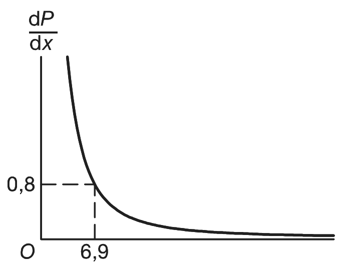
$(1+x)^2 = 62{,}5$

$1 + x = \sqrt{62{,}5} \quad \vee \quad 1 + x = -\sqrt{62{,}5}$

$x = -1 + \sqrt{62{,}5} \quad \vee \quad x = -1 - \sqrt{62{,}5}$

$x \approx 6{,}9$ vold. niet

De snelheid is minder dan 0,8 voor $x > 6{,}9$.



bladzijde 112

11 **a** $P = \dfrac{100(t^2 - t + 1)}{t^2 + 1}$ geeft

$$\frac{dP}{dt} = \frac{(t^2+1)\cdot 100(2t-1) - 100(t^2 - t + 1)\cdot 2t}{(t^2+1)^2}$$

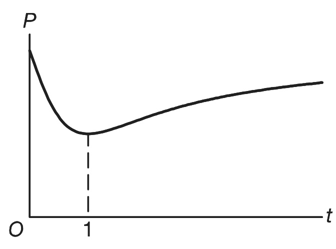
$$= \frac{200t^3 - 100t^2 + 200t - 100 - 200t^3 + 200t^2 - 200t}{(t^2+1)^2} = \frac{100t^2 - 100}{(t^2+1)^2}$$

$\left[\dfrac{dP}{dt}\right]_{t=4} = \dfrac{100\cdot 16 - 100}{(16+1)^2} = \dfrac{1500}{17^2} \approx 5{,}19\%$ per week

De snelheid waarmee P na 4 weken toeneemt is $\dfrac{5{,}19}{7} = 0{,}7\%$ per dag.

b $t = 1$ geeft $\dfrac{dP}{dt} = \dfrac{100\cdot 1^2 - 100}{(100^2+1)^2} = 0$

Uit de schets van de grafiek van P volgt dat het zuurstofgehalte na 1 week minimaal is.



c $P = 98$ geeft $\dfrac{100(t^2 - t + 1)}{t^2 + 1} = 98$

Voer in $y_1 = \dfrac{100(x^2 - x + 1)}{x^2 + 1}$ en $y_2 = 98$.

De optie intersect geeft $x \approx 0{,}02$ en $x \approx 49{,}98$.

Dus na $49{,}98 \cdot 7 \approx 350$ dagen.

d $\left[\dfrac{\mathrm{d}P}{\mathrm{d}t}\right]_{t=8} \approx 1{,}49\%$ per week

$P(8) \approx 87{,}69$

Het zuurstofgehalte moet na 8 weken nog $100 - 87{,}69 = 12{,}31\%$ stijgen.

Met een snelheid van 1,49% per week duurt dit nog $\dfrac{12{,}31}{1{,}49} \cdot 7 \approx 58$ dagen.

Dus vanaf het begin duurt het $8 \cdot 7 + 58 = 114$ dagen voordat het oorspronkelijke zuurstofniveau bereikt wordt.

12 **a** $C = \dfrac{0{,}16t}{t^2 + 4t + 4}$ geeft

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{(t^2 + 4t + 4) \cdot 0{,}16 - 0{,}16t \cdot (2t + 4)}{(t^2 + 4t + 4)^2}$$

$$= \frac{0{,}16t^2 + 0{,}64t + 0{,}64 - 0{,}32t^2 - 0{,}64t}{(t^2 + 4t + 4)^2} = \frac{-0{,}16t^2 + 0{,}64}{(t^2 + 4t + 4)^2}$$

$$\left[\frac{\mathrm{d}C}{\mathrm{d}t}\right]_{t=0} = \frac{0{,}64}{4^2} = 0{,}04 > 0$$

Dus de concentratie neemt direct na toediening toe.

b $\dfrac{\mathrm{d}C}{\mathrm{d}t} = 0$ geeft $\dfrac{-0{,}16t^2 + 0{,}64}{(t^2 + 4t + 4)^2} = 0$

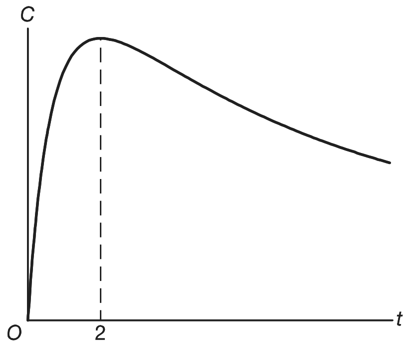
$-0{,}16t^2 + 0{,}64 = 0$

$-0{,}16t^2 = -0{,}64$

$t^2 = 4$

$t = 2 \lor t = -2$

Uit de schets blijkt dat na 2 uur de concentratie maximaal is.



c $\left[\dfrac{\mathrm{d}C}{\mathrm{d}t}\right]_{t=5} \approx -0{,}0014$

$\left[\dfrac{\mathrm{d}C}{\mathrm{d}t}\right]_{t=10} \approx -0{,}00074$

Je krijgt $\dfrac{-0{,}0014}{-0{,}00074} \approx 1{,}9$, dus de snelheid op $t = 5$ is ongeveer 2 keer zo groot als de snelheid op $t = 10$.

d $t = 24$ geeft $C \approx 0{,}0057 > 0{,}005$, dus na 24 uur is de aanwezigheid nog aantoonbaar.

13 **a** Het vierde uur is van $t = 3$ tot $t = 4$.

$t = 3$ geeft $T \approx 38{,}71$

$t = 4$ geeft $T \approx 39{,}09$

De temperatuur neemt met $39{,}09 - 38{,}71 = 0{,}38$ °C toe.

b $\dfrac{\mathrm{d}T}{\mathrm{d}t} = \dfrac{(t^2 + 70) \cdot 45 - 45t \cdot 2t}{(t^2 + 70)^2} = \dfrac{45t^2 + 3150 - 90t^2}{(t^2 + 70)^2} = \dfrac{-45t^2 + 3150}{(t^2 + 70)^2}$

Bij 2 mei om 17.30 uur hoort $t = 29{,}5$.

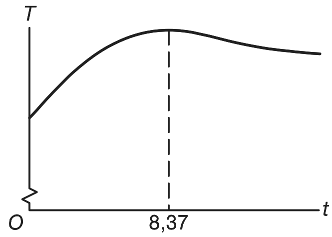
$$\left[\frac{\mathrm{d}T}{\mathrm{d}t}\right]_{t=29{,}5} = \frac{-45 \cdot 29{,}5^2 + 3150}{(29{,}5^2 + 70)^2} \approx -0{,}04$$

De snelheid waarmee de temperatuur afneemt is 0,04 °C per uur.

c $\frac{dT}{dt} = 0$ geeft $\frac{-45t^2 + 3150}{(t^2+70)^2} = 0$

$-45t^2 + 3150 = 0$
$-45t^2 = -3150$
$t^2 = 70$
$t \approx -8{,}37 \ \vee \ t \approx 8{,}37$

Uit de schets van de grafiek van T volgt dat de temperatuur van Frank maximaal is voor $t \approx 8{,}37$.
De maximale temperatuur is ongeveer 39,7 °C.



12.2 Oppervlakten en afstanden bij grafieken

bladzijde 114

14 **a** $PQ = y_Q = f(2) = \sqrt{5 - 2 \cdot 2} = 1$

$O(OPQR) = OP \cdot PQ = 2 \cdot 1 = 2$

b $PQ = y_Q = f(1) = \sqrt{5 - 2 \cdot 1} = \sqrt{3}$

$O(OPQR) = OP \cdot PQ = 1 \cdot \sqrt{3} = \sqrt{3}$

c $PQ = y_Q = f(p) = \sqrt{5 - 2p}$

$A = OP \cdot PQ = p\sqrt{5 - 2p}$

bladzijde 115

15 **a** $PQ = y_P = f(p) = \sqrt{3 - p}$

In $\triangle OQP$ is $OQ^2 + PQ^2 = OP^2$

$p^2 + (\sqrt{3 - p})^2 = OP^2$

$p^2 + 3 - p = OP^2$

Dus $L = OP = \sqrt{p^2 - p + 3}$.

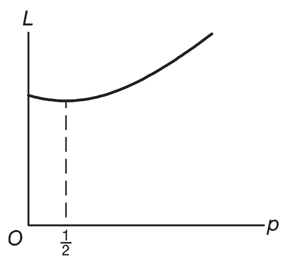
b $L = \sqrt{p^2 - p + 3} = (p^2 - p + 3)^{\frac{1}{2}}$ geeft $\frac{dL}{dp} = \frac{1}{2}(p^2 - p + 3)^{-\frac{1}{2}} \cdot (2p - 1) = \frac{2p - 1}{2\sqrt{p^2 - p + 3}}$

$\frac{dL}{dp} = 0$ geeft $\frac{2p - 1}{2\sqrt{p^2 - p + 3}} = 0$

$2p - 1 = 0$

$2p = 1$

$p = \frac{1}{2}$



De minimale lengte van OP is $\sqrt{(\frac{1}{2})^2 - \frac{1}{2} + 3} = \sqrt{\frac{11}{4}} = \frac{1}{2}\sqrt{11}$.

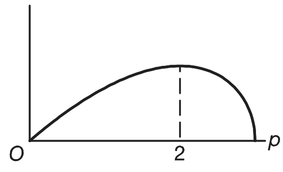
c $A = \frac{1}{2} \cdot OQ \cdot PQ = \frac{1}{2}p\sqrt{3 - p}$

d $A = \frac{1}{2}p\sqrt{3 - p} = \frac{1}{2}p(3 - p)^{\frac{1}{2}}$ geeft

$\frac{dA}{dp} = \frac{1}{2}\sqrt{3 - p} + \frac{1}{2}p \cdot \frac{1}{2}(3 - p)^{-\frac{1}{2}} \cdot -1 = \frac{1}{2}\sqrt{3 - p} - \frac{p}{4\sqrt{3 - p}}$

$\frac{dA}{dp} = 0$ geeft $\frac{1}{2}\sqrt{3 - p} = \frac{p}{4\sqrt{3 - p}}$

$p = 2(3 - p)$
$p = 6 - 2p$
$3p = 6$
$p = 2$

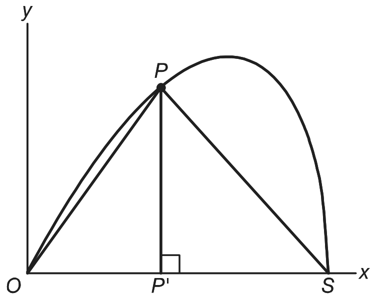


De maximale waarde van A is $\frac{1}{2} \cdot 2 \cdot \sqrt{3 - 2} = 1$.

16 **a** $O(\triangle OSP) = \frac{1}{2} \cdot OS \cdot \text{hoogte}$

$= \frac{1}{2} \cdot OS \cdot PP'$

en PP' is maximaal als P de top van de grafiek is.
De oppervlakte is dus maximaal voor $p = x_{\text{top}}$.



b Bereken y_{top}.

$f(x) = x\sqrt{8-2x}$ geeft

$f'(x) = 1 \cdot \sqrt{8-2x} + x \cdot \dfrac{1}{2\sqrt{8-2x}} \cdot -2$

$= \sqrt{8-2x} - \dfrac{x}{\sqrt{8-2x}}$

$f'(x) = 0$ geeft $\sqrt{8-2x} - \dfrac{x}{\sqrt{8-2x}} = 0$

$\dfrac{\sqrt{8-2x}}{1} = \dfrac{x}{\sqrt{8-2x}}$

$x = 8 - 2x$

$3x = 8$

$x = \frac{8}{3}$

$y_{\text{top}} = f(\frac{8}{3}) = \frac{8}{3}\sqrt{8 - 2 \cdot \frac{8}{3}} = \frac{8}{3}\sqrt{\frac{8}{3}}$

De oppervlakte is maximaal $\frac{1}{2} \cdot OS \cdot y_{\text{top}} = \frac{1}{2} \cdot 4 \cdot \frac{8}{3}\sqrt{\frac{8}{3}} = \frac{16}{3}\sqrt{\frac{8}{3}}$.

c $A = \frac{1}{2} \cdot QS \cdot QP = \frac{1}{2} \cdot (4 - p) \cdot f(p)$

$= \frac{1}{2}(4 - p) \cdot p\sqrt{8-2p} = (2p - \frac{1}{2}p^2)\sqrt{8-2p}$

d $\dfrac{dA}{dp} = (2 - p)\sqrt{8-2p} + (2p - \frac{1}{2}p^2) \cdot \dfrac{1}{2\sqrt{8-2p}} \cdot -2$

$= (2 - p)\sqrt{8-2p} - \dfrac{2p - \frac{1}{2}p^2}{\sqrt{8-2p}}$

e $\left[\dfrac{dA}{dp}\right]_{p=1,6} = (2-1,6)\sqrt{8-2\cdot 1,6} - \dfrac{2\cdot 1,6 - \frac{1}{2}\cdot 1,6^2}{\sqrt{8-2\cdot 1,6}} = 0,4\sqrt{4,8} - \dfrac{1,92}{\sqrt{4,8}} = 0$

Dus Sandra heeft gelijk.

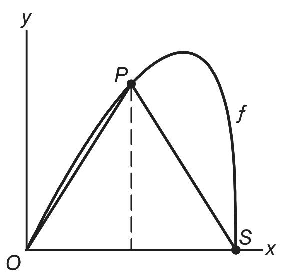
17 **a** Stel $x_P = p$. Dit geeft $y_P = p \cdot \sqrt[3]{8-p}$.

$f(x) = 0$ geeft $x \cdot \sqrt[3]{8-x} = 0$

$x = 0 \lor \sqrt[3]{8-x} = 0$

$x = 0 \lor 8 - x = 0$

$x = 0 \lor x = 8$, dus $x_S = 8$



$A = O(\triangle OSP) = \frac{1}{2} \cdot x_S \cdot y_P = \frac{1}{2} \cdot 8 \cdot p \cdot \sqrt[3]{8-p}$

$= 4p \cdot \sqrt[3]{8-p} = 4p(8-p)^{\frac{1}{3}}$

$\dfrac{dA}{dp} = 4 \cdot \sqrt[3]{8-p} + 4p \cdot \frac{1}{3}(8-p)^{-\frac{2}{3}} \cdot -1 = 4 \cdot \sqrt[3]{8-p} - \dfrac{4p}{3 \cdot \sqrt[3]{(8-p)^2}}$

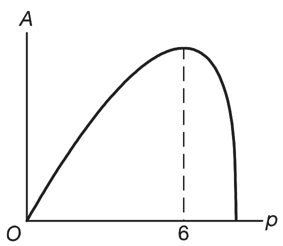
$\dfrac{dA}{dp} = 0$ geeft $4 \cdot \sqrt[3]{8-p} = \dfrac{4p}{3 \cdot \sqrt[3]{(8-p)^2}}$

$12(8 - p) = 4p$

$96 - 12p = 4p$

$-16p = -96$

$p = 6$



De maximale oppervlakte van $\triangle OSP$ is
$4 \cdot 6 \cdot \sqrt[3]{8-6} = 24 \cdot \sqrt[3]{2}$.

b $A = O(\triangle OPQ) = \frac{1}{2} \cdot x_P \cdot y_P = \frac{1}{2} p \cdot p \cdot \sqrt[3]{8-p} = \frac{1}{2} p^2 \cdot \sqrt[3]{8-p}$

c $A = \frac{1}{2} p^2 \cdot \sqrt[3]{8-p} = \frac{1}{2} p^2 (8-p)^{\frac{1}{3}}$ geeft

$$\frac{dA}{dp} = p \cdot \sqrt[3]{8-p} + \frac{1}{2} p^2 \cdot \frac{1}{3}(8-p)^{-\frac{2}{3}} \cdot -1 = p \cdot \sqrt[3]{8-p} - \frac{p^2}{6 \cdot \sqrt[3]{(8-p)^2}}$$

$$\left[\frac{dA}{dp}\right]_{p=7} = 7 \cdot \sqrt[3]{8-7} - \frac{7^2}{6 \cdot \sqrt[3]{(8-7)^2}} = 7 - \frac{49}{6} = -1\frac{1}{6}$$

Omdat $\left[\frac{dA}{dp}\right]_{p=7} \neq 0$, is A niet maximaal voor $p = 7$.

18 **a** $y_P = 3 - \frac{1}{2} p^2$

$A = O(\triangle OPQ) = \frac{1}{2} \cdot PQ \cdot y_P = \frac{1}{2} \cdot 2p \cdot (3 - \frac{1}{2} p^2) = 3p - \frac{1}{2} p^3$

b $\frac{dA}{dp} = 3 - 1\frac{1}{2} p^2$

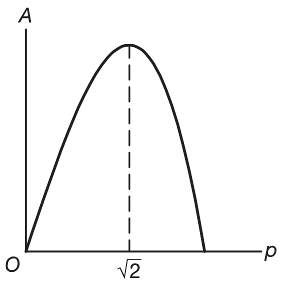
$\frac{dA}{dp} = 0$ geeft $3 - 1\frac{1}{2} p^2$

$-1\frac{1}{2} p^2 = -3$

$p^2 = 2$, dus $p = \sqrt{2}$

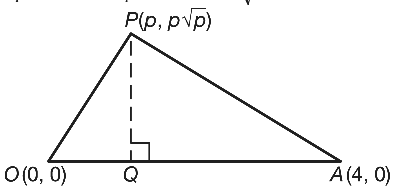
De maximale oppervlakte van $\triangle OPQ$ is

$3\sqrt{2} - \frac{1}{2}(\sqrt{2})^3 = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$.



bladzijde 117

19 **a** $x_P = p$, dus $y_P = f(p) = p\sqrt{p}$



$PQ = p\sqrt{p}$ en $AQ = 4 - p$

In $\triangle APQ$ is $PQ^2 + AQ^2 = AP^2$

$(p\sqrt{p})^2 + (4-p)^2 = AP^2$

$p^3 + 16 - 8p + p^2 = AP^2$

$AP^2 = p^3 + p^2 - 8p + 16$, dus

$AP = \sqrt{p^3 + p^2 - 8p + 16}$

b $L = AP = \sqrt{p^3 + p^2 - 8p + 16} = (p^3 + p^2 - 8p + 16)^{\frac{1}{2}}$

$$\frac{dL}{dp} = \frac{1}{2}(p^3 + p^2 - 8p + 16)^{-\frac{1}{2}} \cdot (3p^2 + 2p - 8) = \frac{3p^2 + 2p - 8}{2\sqrt{p^3 + p^2 - 8p + 16}}$$

$\frac{dL}{dp} = 0$ geeft $\frac{3p^2 + 2p - 8}{2\sqrt{p^3 + 2p - 8p + 16}} = 0$

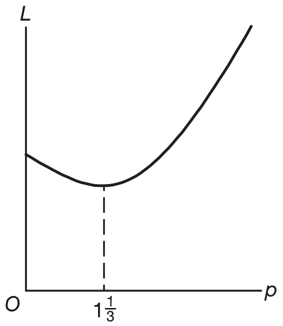
$3p^2 + 2p - 8 = 0$

$D = 2^2 - 4 \cdot 3 \cdot -8 = 100$

$p = \frac{-2-10}{6} = -2 \vee p = \frac{-2+10}{6} = 1\frac{1}{3}$

Dus $p = 1\frac{1}{3}$.

Dus het punt op de grafiek dat het dichtst bij A ligt is $(1\frac{1}{3}, \frac{1}{3}\sqrt{1\frac{1}{3}})$.



20 Stel $x_B = p$, dus $y_B = f(p) = p^2 - 2$.
$OC = p^2 - 2$, $AC = 6 - (p^2 - 2) = 8 - p^2$ en $BC = p$
In $\triangle OBC$ is $OC^2 + BC^2 = OB^2$

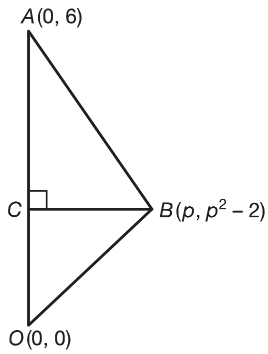
$$(p^2 - 2)^2 + p^2 = OB^2$$
$$p^4 - 4p^2 + 4 + p^2 = OB^2$$
$$OB^2 = p^4 - 3p^2 + 4, \text{ dus } OB = \sqrt{p^4 - 3p^2 + 4}$$

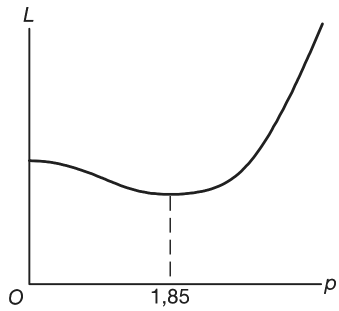
In $\triangle ABC$ is $AC^2 + BC^2 = AB^2$

$$(8 - p^2)^2 + p^2 = AB^2$$
$$64 - 16p^2 + p^4 + p^2 = AB^2$$
$$AB^2 = p^4 - 15p^2 + 64, \text{ dus } AB = \sqrt{p^4 - 15p^2 + 64}$$

$$L = OB + AB = \sqrt{p^4 - 3p^2 + 4} + \sqrt{p^4 - 15p^2 + 64}$$

Voer in $y_1 = \sqrt{x^4 - 3x^2 + 4} + \sqrt{x^4 - 15x^2 + 64}$.
De optie minimum geeft $x \approx 1{,}85$ en $y \approx 7{,}27$.
De minimale waarde van L is ongeveer 7,27.





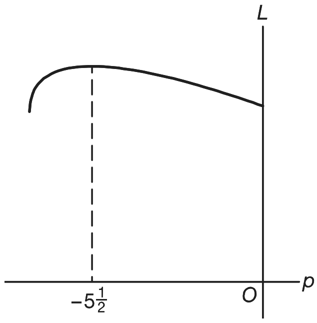
21 **a** $L = y_A - y_B = f(-3) - g(-3) = \sqrt{2 \cdot -3 + 15} - \frac{1}{2} \cdot -3 = 4\frac{1}{2}$

b $L = y_A - y_B = f(p) - g(p) = \sqrt{2p + 15} - \frac{1}{2}p$

c $L = \sqrt{2p + 15} - \frac{1}{2}p = (2p + 15)^{\frac{1}{2}} - \frac{1}{2}p$ geeft

$$\frac{dL}{dp} = \frac{1}{2}(2p + 15)^{-\frac{1}{2}} \cdot 2 - \frac{1}{2} = \frac{1}{\sqrt{2p + 15}} - \frac{1}{2}$$

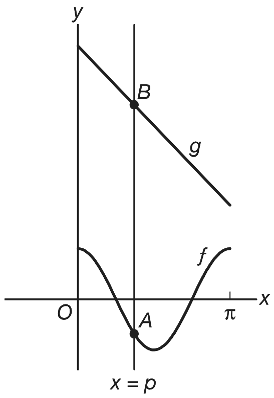
$$\frac{dL}{dp} = 0 \text{ geeft } \frac{1}{\sqrt{2p + 15}} = \frac{1}{2}$$
$$\sqrt{2p + 15} = 2$$
$$2p + 15 = 4$$
$$2p = -11$$
$$p = -5\tfrac{1}{2}$$



De lengte van AB is maximaal voor $p = -5\frac{1}{2}$.

bladzijde 118

22



Voor de lengte L van AB geldt $L = g(p) - f(p) = 5 - p - \cos(2p)$.

$$\frac{dL}{dp} = -1 + 2\sin(2p)$$

$$\frac{dL}{dp} = 0 \text{ geeft } -1 + 2\sin(2p) = 0$$

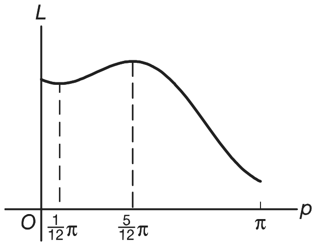
$$2\sin(2p) = 1$$

$$\sin(2p) = \tfrac{1}{2}$$

$$2p = \tfrac{1}{6}\pi + k \cdot 2\pi \ \vee \ 2p = \tfrac{5}{6}\pi + k \cdot 2\pi$$

$$p = \tfrac{1}{12}\pi + k \cdot \pi \ \vee \ p = \tfrac{5}{12}p + k \cdot \pi$$

p op $[0, \pi]$ geeft $p = \frac{1}{12}\pi \ \vee \ p = \frac{5}{12}\pi$.



De lengte van AB is maximaal voor $p = \frac{5}{12}\pi$.

bladzijde 119

23 **a** Uit de figuur blijkt dat voor de lengte L van AB geldt

$L = f(p) - g(p) = \sqrt{6p + 12} - (p + 2) = \sqrt{6p + 12} - p - 2$.

b $L = \sqrt{6p + 12} - p - 2 = (6p + 12)^{\frac{1}{2}} - p - 2$ geeft $\dfrac{dL}{dp} = \frac{1}{2}(6p + 12)^{-\frac{1}{2}} \cdot 6 - 1 = \dfrac{3}{\sqrt{6p + 12}} - 1$

$$\frac{dL}{dp} = 0 \text{ geeft } \frac{3}{\sqrt{6p + 12}} = 1$$

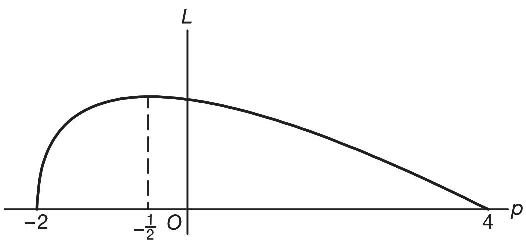
$$\sqrt{6p + 12} = 3$$

$$6p + 12 = 9$$

$$6p = -3$$

$$p = -\tfrac{1}{2}$$

De maximale waarde van L is $\sqrt{6 \cdot -\frac{1}{2} + 12} - -\frac{1}{2} - 2 = 1\frac{1}{2}$.



24 Uit de figuur blijkt dat voor de lengte L van CD geldt

$$L = \tfrac{1}{2}p + 5 - \sqrt{p^2 + 4} = \tfrac{1}{2}p + 5 - (p^2 + 4)^{\frac{1}{2}}.$$

$$\frac{dL}{dp} = \tfrac{1}{2} - \tfrac{1}{2}(p^2 + 4)^{-\frac{1}{2}} \cdot 2p = \tfrac{1}{2} - \frac{p}{\sqrt{p^2 + 4}}$$

$\frac{dL}{dp} = 0$ geeft $\frac{p}{\sqrt{p^2+4}} = \frac{1}{2}$

$\sqrt{p^2+4} = 2p$

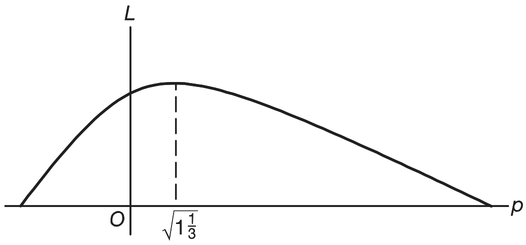
$p^2 + 4 = 4p^2$

$3p^2 = 4$

$p^2 = 1\frac{1}{3}$

$p = -\sqrt{1\frac{1}{3}} \vee p = \sqrt{1\frac{1}{3}}$

voldoet niet voldoet



De maximale lengte van CD is $\frac{1}{2}\sqrt{1\frac{1}{3}} + 5 - \sqrt{(\sqrt{1\frac{1}{3}})^2 + 4} \approx 3{,}27$.

25 **a** De lengte van AB is

$L = f(p) - g(p)$

$= \sqrt{25-p^2} - (\frac{3}{4}p - 4)$

$= \sqrt{25-p^2} - \frac{3}{4}p + 4$

$\frac{dL}{dp} = \frac{1}{2\sqrt{25-p^2}} \cdot -2p - \frac{3}{4} = -\frac{p}{\sqrt{25-p^2}} - \frac{3}{4}$

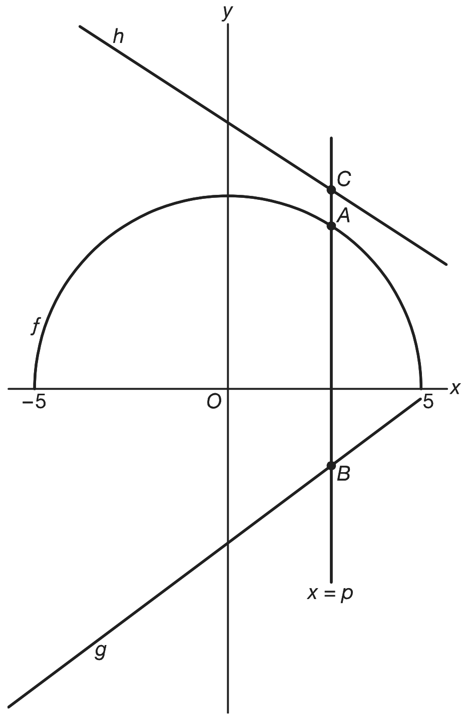
Voer in $y_1 = -\frac{x}{\sqrt{25-x^2}} - \frac{3}{4}$.

De optie zero (TI) of Root (Casio) geeft $x = -3$.

$L(3) = \sqrt{25-(-3)^2} - (\frac{3}{4} \cdot -3 - 4) = 10\frac{1}{4}$

Zie de schets.

Dus AB is maximaal $10\frac{1}{4}$ voor $p = -3$.



b De lengte van AC is

$L = h(p) - f(p) = -\frac{4}{3}p + 10 - \sqrt{25-p^2}$

$\frac{dL}{dp} = -\frac{4}{3} - \frac{1}{2\sqrt{25-p^2}} \cdot -2p = -\frac{4}{3} + \frac{p}{\sqrt{25-p^2}}$

Voer in $y_1 = -\frac{4}{3} + \frac{x}{\sqrt{25-x^2}}$.

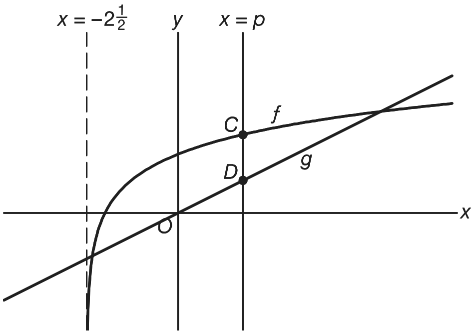
De optie zero (TI) of ROOT (Casio) geeft $x = 4$.

$L(4) = -\frac{4}{3} \cdot 4 + 10 - \sqrt{25-4^2} = 1\frac{2}{3}$

Zie de schets.

Dus AC is minimaal $1\frac{2}{3}$ voor $p = 4$.

26



Uit de schets blijkt dat voor de lengte L van CD geldt

$L = f(p) - g(p) = \ln(2p+5) - \frac{1}{2}p$.

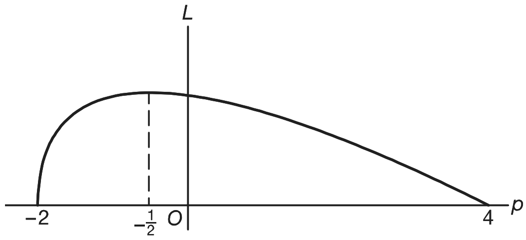
$\frac{dL}{dp} = \frac{1}{2p+5} \cdot 2 - \frac{1}{2} = \frac{2}{2p+5} - \frac{1}{2}$

$\frac{dL}{dp} = 0$ geeft $\frac{2}{2p+5} = \frac{1}{2}$

$2p + 5 = 4$

$2p = -1$

$p = -\frac{1}{2}$



De maximale lengte van CD is $\ln(2 \cdot -\frac{1}{2} + 5) - \frac{1}{2} \cdot -\frac{1}{2} = \ln(4) + \frac{1}{4}$.

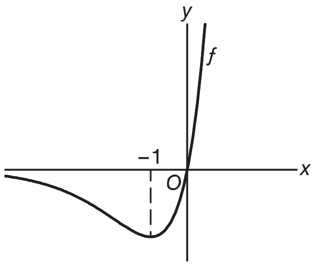
27 **a** $f(x) = 5x\,e^x$ geeft $f'(x) = 5\,e^x + 5x\,e^x = (5+5x)e^x$

$f'(x) = 0$ geeft $(5+5x)e^x = 0$

$5 + 5x = 0$

$5x = -5$

$x = -1$



min. is $f(-1) = -5e^{-1} = -\frac{5}{e}$

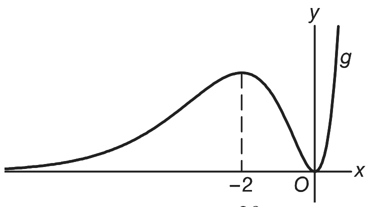
b $g(x) = 5x^2\,e^x$ geeft $g'(x) = 10x\,e^x + 5x^2\,e^x = (10x + 5x^2)e^x$

$g'(x) = 0$ geeft $(10x + 5x^2)e^x = 0$

$10x + 5x^2 = 0$

$5x(2 + x) = 0$

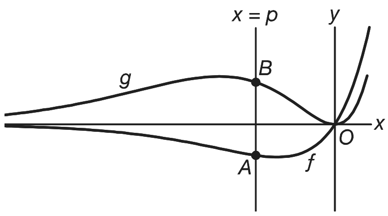
$x = 0 \lor x = -2$



max. is $g(-2) = 20e^{-2} = \frac{20}{e^2}$

min. is $g(0) = 0$

c



Voor de lengte L van AB geldt $L = g(p) - f(p) = 5p^2\,e^p - 5p\,e^p = (5p^2 - 5p)e^p$.

$\frac{dL}{dp} = (10p - 5)e^p + (5p^2 - 5p)e^p = (5p^2 + 5p - 5)e^p$

$\frac{dL}{dp} = 0$ geeft $(5p^2 + 5p - 5)e^p = 0$

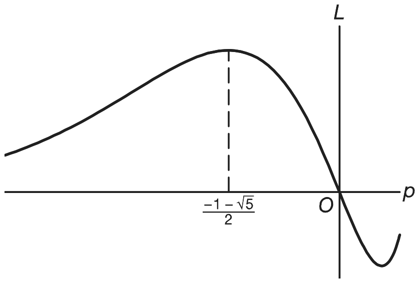
$5p^2 + 5p - 5 = 0$

$p^2 + p - 1 = 0$

$D = 1^2 - 4 \cdot 1 \cdot -1 = 5$

$p = \frac{-1-\sqrt{5}}{2} \lor p = \frac{-1+\sqrt{5}}{2}$

vold. niet



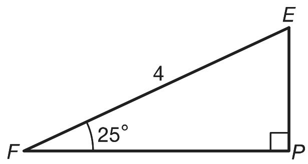
De lengte van AB is maximaal voor $p = \dfrac{-1-\sqrt{5}}{2}$.

12.3 Goniometrische modellen

bladzijde 121

28 **a** In $\triangle EFP$: $\sin 25° = \dfrac{EP}{4}$

$EP = 4 \sin 25° \approx 1{,}690$



b In $\triangle EFP$: $\cos 25° = \dfrac{FP}{4}$

$FP = 4 \cos 25° \approx 3{,}625$

c

$O = 4 \cdot O(\triangle EPF) + 2 \cdot O(PQDE)$

$= 4 \cdot \frac{1}{2} \cdot FP \cdot EP + 2 \cdot PQ \cdot EP$

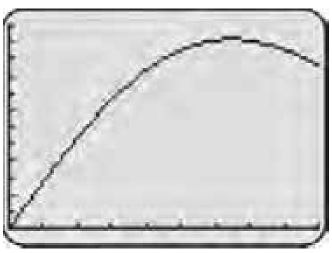
$= 4 \cdot \frac{1}{2} \cdot 3{,}625 \cdot 1{,}690 + 2 \cdot 6 \cdot 1{,}690 \approx 32{,}54$

d $EP = 4 \sin 40° \approx 2{,}571$
$FP = 4 \cos 40° \approx 3{,}064$
$O = 4 \cdot \frac{1}{2} \cdot FP \cdot EP + 2 \cdot PQ \cdot EP$

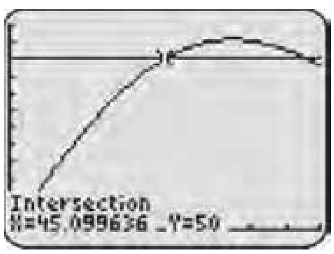
$= 4 \cdot \frac{1}{2} \cdot 3{,}064 \cdot 2{,}571 + 2 \cdot 6 \cdot 2{,}571 \approx 46{,}61$

bladzijde 122

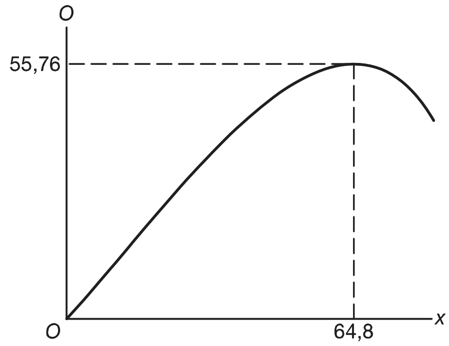
29 **a**



b Voer in $y_2 = 50$.
De optie intersect geeft $x \approx 45$ en $x \approx 86$.
Dus $O = 50$ voor $\alpha \approx 45°$ en voor $\alpha \approx 86°$.

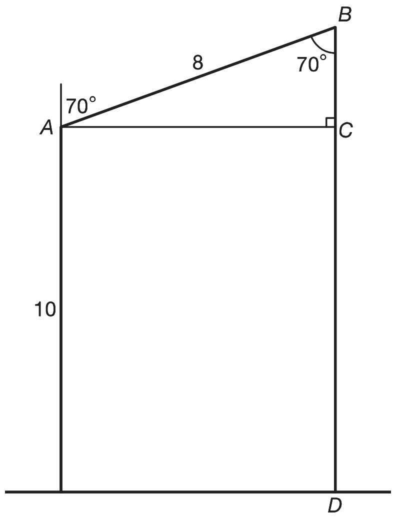


c



De optie maximum geeft $x \approx 64{,}8$ en $y \approx 55{,}76$.
Dus bij $\alpha \approx 65°$ is de oppervlakte $O_{max} \approx 55{,}76$.

30 **a** Trek $AC \perp BD$.



In $\triangle ACB$: $\cos 70° = \dfrac{BC}{8}$

$BC = 8 \cdot \cos 70° \approx 2{,}736$

Dit geeft $h = 10 + BC \approx 12{,}74$ m.

b In $\triangle ACB$ is $\angle B = \alpha$ (Z-hoeken)

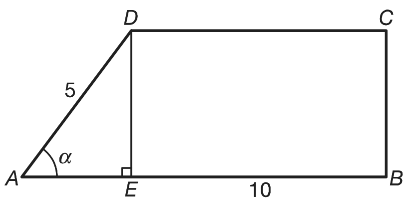
$\cos(\alpha) = \dfrac{BC}{8}$, dus $BC = 8\cos(\alpha)$

Dit geeft $h = 10 + 8\cos(\alpha)$.

c Voer in $y_1 = 10 + 8\cos(x)$ en $y_2 = 15$.
Kies bijvoorbeeld Xmin = 0, Xmax = 90, Ymin = 0 en Ymax = 20
De optie intersect geeft $x \approx 51{,}3$.
Dus voor een hoek van 51° is $h = 15$ m.

bladzijde 123

31 **a**



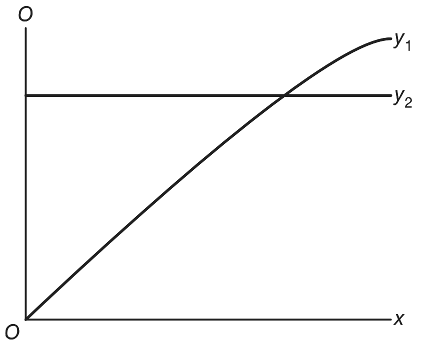
Trek $DE \perp AB$.

In $\triangle AED$: $\sin(\alpha) = \dfrac{ED}{5}$, dus $ED = 5\sin(\alpha)$

$$\left.\begin{array}{l} \text{en } \cos(\alpha) = \dfrac{AE}{5}, \text{ dus } AE = 5\cos(\alpha) \\ AB = 10 \end{array}\right\} EB = 10 - 5\cos(\alpha)$$

$$\begin{aligned} O(ABCD) &= O(AED) + O(EBCD) \\ &= \tfrac{1}{2} \cdot AE \cdot ED + EB \cdot ED \\ &= \tfrac{1}{2} \cdot 5\cos(\alpha) \cdot 5\sin(\alpha) + (10 - 5\cos(\alpha)) \cdot 5\sin(\alpha) \\ &= 12\tfrac{1}{2}\cos(\alpha)\sin(\alpha) + 50\sin(\alpha) - 25\cos(\alpha)\sin(\alpha) \\ &= 50\sin(\alpha) - 12\tfrac{1}{2}\cos(\alpha)\sin(\alpha) \\ &= 50\sin(\alpha) - 12\tfrac{1}{2}\sin(\alpha)\cos(\alpha) \end{aligned}$$

b Voer in $y_1 = 50 \sin(x) - 12{,}5 \sin(x) \cos(x)$ en $y_2 = 40$.
Kies bijvoorbeeld Xmin = 0, Xmax = 90, Ymin = 0 en Ymax = 50



De optie intersect geeft $x \approx 63{,}97$.
Dus voor $\alpha \geq 64°$ is $O > 40$.

32 **a** AE en BF loodrecht op CD.
$\angle ADE = \alpha$ (Z-hoeken)

In $\triangle AED$: $\sin(\alpha) = \dfrac{AE}{10}$, dus $AE = 10 \sin(\alpha)$

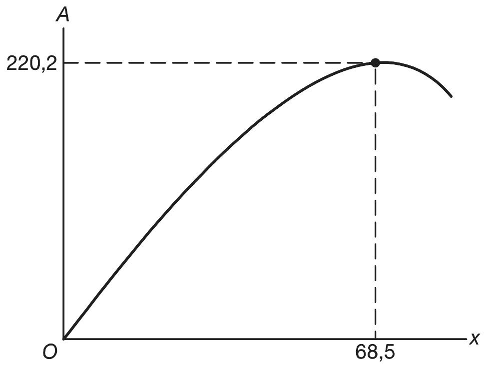
en $\cos(\alpha) = \dfrac{DE}{10}$, dus $DE = 10 \cos(\alpha)$.

$A = O(\text{dwarsdoorsnede}) = O(ABFE) + 2 \cdot O(AED)$

$= AB \cdot AE + 2 \cdot \frac{1}{2} \cdot DE \cdot AE$

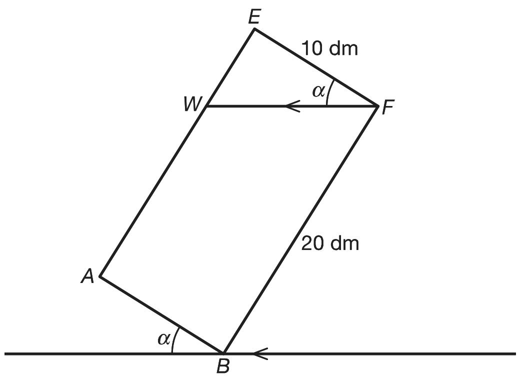
$= 20 \cdot 10 \sin(\alpha) + 10 \cos(\alpha) \cdot 10 \sin(\alpha)$
$= 200 \sin(\alpha) + 100 \sin(\alpha) \cos(\alpha)$

b Voer in $y_1 = 200 \sin(x) + 100 \sin(x) \cos(x)$.



De optie maximum geeft $x \approx 68{,}53$ en $y \approx 220{,}2$.
Dus bij $\alpha \approx 69°$ is $A_{max} \approx 220$ cm^2.

33 **a**



De bak heeft een inhoud van $20 \cdot 10 \cdot 10 = 2000\ \text{dm}^3 = 2000$ liter.
Bij kantelhoek α staat de waterspiegel volgens de horizontale lijn WF met $\angle EFW = \alpha$.

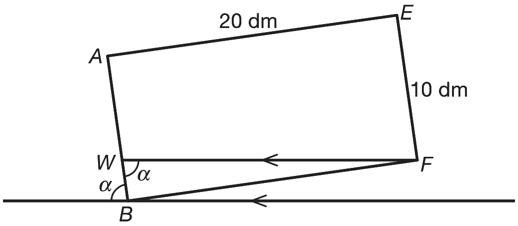
In $\triangle EFW$: $\tan(\alpha) = \dfrac{EW}{EF} = \dfrac{EW}{10}$, dus $EW = 10\tan(\alpha)$ dm.

De hoeveelheid weggestroomd water is precies de inhoud van het prisma met grondvlak $\triangle EFW$ en hoogte $FG = 10$ dm.

$I_{\text{prisma}} = \frac{1}{2} \cdot EW \cdot EF \cdot FG = \frac{1}{2} \cdot 10\tan(\alpha) \cdot 10 \cdot 10 = 500\tan(\alpha)$ liter.

Dus $V = 2000 - 500\tan(\alpha)$ liter.

b In de formule $V = 2000 - 500\tan(\alpha)$ is 2000 de inhoud van de hele bak en $500\tan(\alpha)$ de hoeveelheid water die eruit gestroomd is, waarbij dit een driezijdig prisma betreft. In situaties zoals in figuur 12.17d is de inhoud van een vierzijdig prisma weggestroomd. In zo'n situatie kun je rechtstreeks berekenen hoeveel liter water er over is.



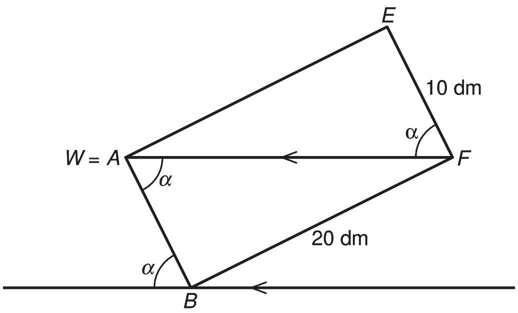
Ook $\angle BWF = \alpha$ (Z-hoeken).

In $\triangle BWF$: $\tan(\alpha) = \dfrac{BF}{BW} = \dfrac{20}{BW}$, dus $BW = \dfrac{20}{\tan(\alpha)}$ dm.

Dit geeft inhoud overgebleven water is

$V = \frac{1}{2} \cdot BW \cdot BF \cdot BC = \frac{1}{2} \cdot \dfrac{20}{\tan(\alpha)} \cdot 20 \cdot 10 = \dfrac{2000}{\tan(\alpha)}$ liter.

c Beide formules komen op hetzelfde neer als W samenvalt met A.



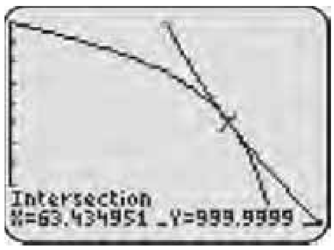
In dit geval is $\tan(\alpha) = \dfrac{BF}{AB} = \dfrac{20}{10} = 2$, dus $\alpha \approx 63{,}4°$.

Alternatieve uitwerking

In het geval beide formules te gebruiken zijn moeten ze dezelfde uitkomst geven.

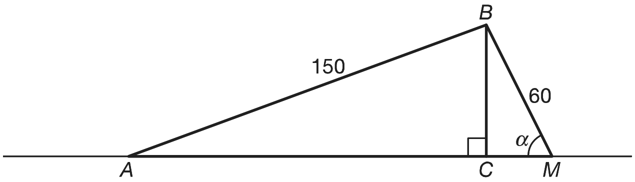
Dus los op: $2000 - 500\tan(\alpha) = \dfrac{2000}{\tan(\alpha)}$.

Voer in $y_1 = 2000 - 500\tan(x)$ en $y_2 = \dfrac{2000}{\tan(x)}$.



De optie intersect geeft $x \approx 63{,}4$ en $y = 1000$.
Dus bij kantelhoek $\alpha \approx 63{,}4°$.

34 **a**



Trek $BC \perp AM$.

In $\triangle CMB$: $\sin(\alpha) = \dfrac{BC}{60}$, dus $BC = 60\sin(\alpha)$

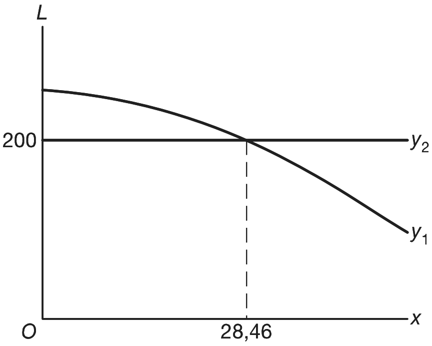
en $\cos(\alpha) = \dfrac{CM}{60}$, dus $CM = 60\cos(\alpha)$.

In $\triangle ABC$ geeft de stelling van Pythagoras
$AC^2 + BC^2 = AB^2$
$AC^2 = 150^2 - (60\sin(\alpha))^2$
$AC^2 = 22\,500 - 3600\sin^2(\alpha)$

dus $AC = \sqrt{22500 - 3600\sin^2(\alpha)}$

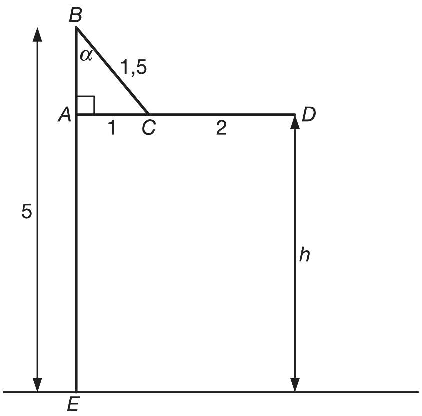
$L = AM = CM + AC = 60\cos(\alpha) + \sqrt{22500 - 3600\sin^2(\alpha)}$

b Voer in $y_1 = 60\cos(x) + \sqrt{22500 - 3600\sin^2(\mathrm{x})}$ en $y_2 = 200$.



De optie intersect geeft $x \approx 28{,}46$.
Dus is $L > 200$ voor $0° < \alpha \leq 28°$.

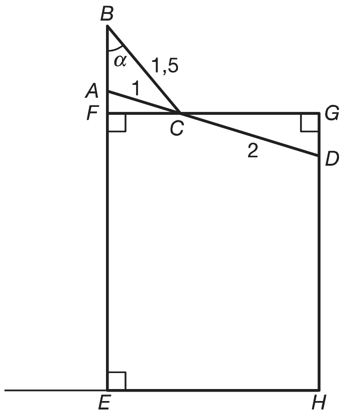
35 **a**



In $\triangle ABC$: $\sin(\alpha) = \dfrac{1}{1{,}5}$

$\alpha \approx 42°$

b Trek de lijn door C loodrecht op EB. F op EB, G op de verticale lijn DH.



In $\triangle FCB$: $\sin(\alpha) = \dfrac{FC}{1,5}$, dus $FC = 1,5\sin(\alpha)$

en $\cos(\alpha) = \dfrac{BF}{1,5}$, dus $BF = 1,5\cos(\alpha)$.

Dit geeft $EF = 5 - 1,5\cos(\alpha)$
In $\triangle ACF$ geeft de stelling van Pythagoras
$AF^2 + FC^2 = 1^2$
$AF^2 = 1 - (1,5\sin(\alpha))^2$
$\quad = 1 - 2,25\sin^2(\alpha)$
dus $AF = \sqrt{1 - 2,25\sin^2(\alpha)}$

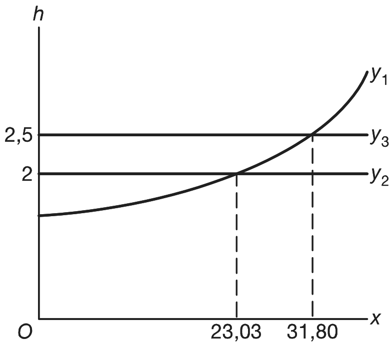
Zandloperfiguur $\triangle ACF \backsim \triangle DCG$.

AC	AF	geeft	1	AF
DC	DG		2	DG

$DG = 2 \cdot AF = 2\sqrt{1 - 2,25\sin^2(\alpha)}$
$h = DH = EF - DG = 5 - 1,5\cos(\alpha) - 2\sqrt{1 - 2,25\sin^2(\alpha)}$

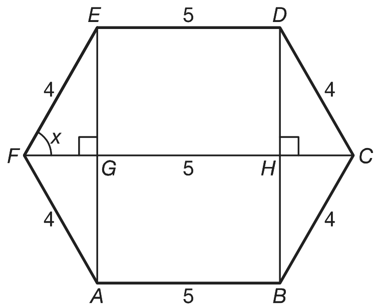
c $h = 5 - 1,5\cos(40°) - 2\sqrt{1 - 2,25\sin^2(40°)} \approx 3,32$ m

d Voer in $y_1 = 5 - 1,5\cos(x) - 2\sqrt{1 - 2,25\sin^2(x)}$, $y_2 = 2$ en $y_3 = 2,5$.
Intersect met y_1 en y_2 geeft $x \approx 23,03$.
Intersect met y_1 en y_3 geeft $x \approx 31,80$.



Dus $2 < h < 2,5$ voor $23° < \alpha < 32°$.

36 **a** In $\triangle FGE$: $\sin(x) = \dfrac{EG}{4}$, dus $EG = 4\sin(x)$



$\cos(x) = \dfrac{FG}{4}$, dus $FG = 4\cos(x)$

$O(\text{zeshoek}) = 4 \cdot O(\triangle FGE) + 2 \cdot O(GHDE)$

$= 4 \cdot \frac{1}{2} \cdot FG \cdot EG + 2 \cdot GH \cdot EG$

$= 2 \cdot 4\cos(x) \cdot 4\sin(x) + 2 \cdot 5 \cdot 4\sin(x)$

$= 32\sin(x)\cos(x) + 40\sin(x)$

b $\dfrac{dO}{dx} = 32\cos(x) \cdot \cos(x) - 32\sin(x) \cdot \sin(x) + 40\cos(x)$

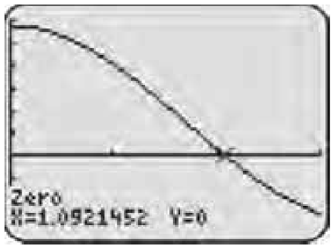
$= 32\cos^2(x) - 32\sin^2(x) + 40\cos(x)$ $\quad \sin^2(x) + \cos^2(x) = 1$

$= 32\cos^2(x) - 32(1 - \cos^2(x)) + 40\cos(x)$ $\quad \sin^2(x) = 1 - \cos^2(x)$

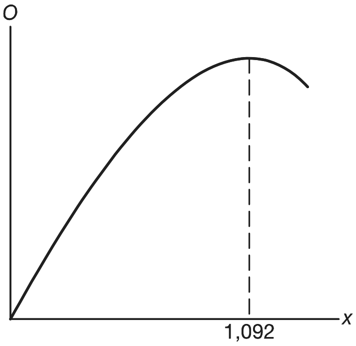
$= 32\cos^2(x) - 32 + 32\cos^2(x) + 40\cos(x)$

$= 64\cos^2(x) + 40\cos(x) - 32$

c Voer in $y_1 = 64\cos^2(x) + 40\cos(x) - 32$.

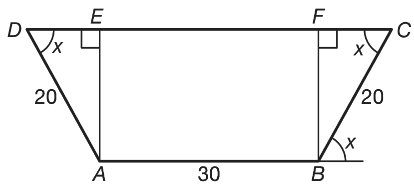


De optie zero (TI) of ROOT (Casio) geeft $x \approx 1{,}092$.



De oppervlakte is maximaal bij een hoek van ongeveer 1,092 rad, dat is $1{,}092 \times \dfrac{180°}{\pi} \approx 63°$.

37 **a** In $\triangle BCF$ is $\angle C = x$.



$\cos(x) = \dfrac{FC}{20}$ dus $FC = 20\cos(x)$

$\sin(x) = \dfrac{BF}{20}$ dus $BF = 20\sin(x)$

$A = O(ABFE) + 2 \cdot O(\triangle BCF)$

$= AB \cdot BF + 2 \cdot \frac{1}{2} \cdot FC \cdot BF$

$= 30 \cdot 20\sin(x) + 20\cos(x) \cdot 20\sin(x)$

$= 600\sin(x) + 400\sin(x)\cos(x)$

b $\dfrac{dA}{dx} = 600\cos(x) + 400\cos(x) \cdot \cos(x) + 400\sin(x) \cdot -\sin(x)$

$= 600\cos(x) + 400\cos^2(x) - 400\sin^2(x)$ $\quad \sin^2(x) + \cos^2(x) = 1$

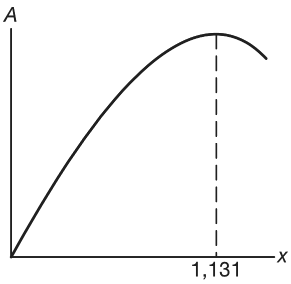
$= 600\cos(x) + 400\cos^2(x) - 400(1 - \cos^2(x))$ $\quad \sin^2(x) = 1 - \cos^2(x)$

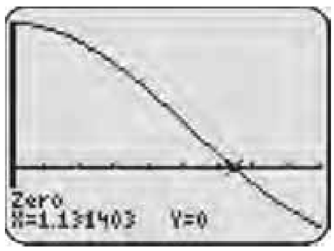
$= 600\cos(x) + 400\cos^2(x) - 400 + 400\cos^2(x)$

$= 800\cos^2(x) + 600\cos(x) - 400$

c Voer in $y_1 = 800\cos^2(x) + 600\cos(x) - 400$.

De optie zero (TI) of ROOT (Casio) geeft $x \approx 1{,}131$.





De oppervlakte is maximaal bij een hoek van ongeveer 1,131 rad, dat is $1{,}131 \times \dfrac{180°}{\pi} \approx 65°$.

38 **a** In $\triangle MNP$: $MN = 7 + 5 = 12$

$$\cos(x) = \frac{MP}{12}, \text{ dus } MP = 12\cos(x)$$

$$\sin(x) = \frac{NP}{12}, \text{ dus } NP = 12\sin(x)$$

In $\triangle ABP$ geeft de stelling van Pythagoras

$$\begin{aligned} AB^2 &= AP^2 + BP^2 \\ &= (7 + 12\cos(x))^2 + (5 + 12\sin(x))^2 \\ &= (7 + 12\cos(x))(7 + 12\cos(x)) + (5 + 12\sin(x))(5 + 12\sin(x)) \\ &= 49 + 84\cos(x) + 84\cos(x) + 144\cos^2(x) + 25 + 60\sin(x) + 60\sin(x) + 144\sin^2(x) \\ &= 49 + 168\cos(x) + 25 + 120\sin(x) + 144(\cos^2(x) + \sin^2(x)) \\ &= 120\sin(x) + 168\cos(x) + 49 + 25 + 144 \cdot 1 \\ &= 120\sin(x) + 168\cos(x) + 218 \end{aligned}$$

Dus $L = AB = \sqrt{120\sin(x) + 168\cos(x) + 218}$.

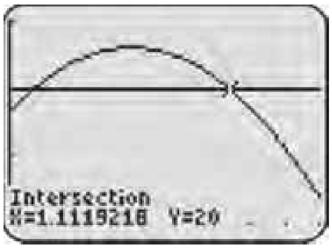
b 25° komt overeen met $x = \frac{25}{180} \cdot \pi \approx 0{,}436$.

$L = \sqrt{120\sin(0{,}436) + 168\cos(0{,}436) + 218} \approx 20{,}5$ dm

c Voer in $y_1 = \sqrt{120\sin(x) + 168\cos(x) + 218}$ en $y_2 = 20$.
De optie intersect geeft $x \approx 0{,}129$ en $x \approx 1{,}112$.

Dat komt overeen met hoeken van $0{,}129 \times \frac{180}{\pi} \approx 7°$ en $1{,}112 \times \frac{180}{\pi} \approx 64°$.

Dus $L = 20$ dm hoort bij $\angle PMN \approx 7°$ en $\angle PMN \approx 64°$.

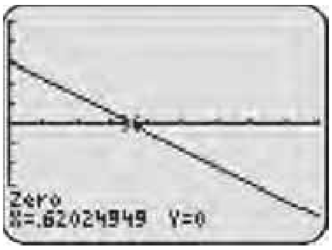


d Stel $L = \sqrt{120\sin(x) + 168\cos(x) + 218} = \sqrt{u}$ met $u = 120\sin(x) + 168\cos(x) + 218$.

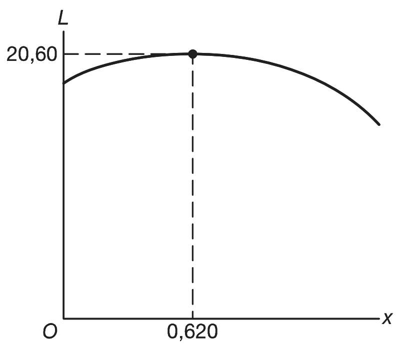
$$\frac{dL}{dx} = \frac{dL}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} \cdot (120\cos(x) - 168\sin(x))$$

$$= \frac{60\cos(x) - 84\sin(x)}{\sqrt{120\sin(x) + 168\cos(x) + 218}}$$

Voer in $y_2 = \dfrac{60\cos(x) - 84\sin(x)}{\sqrt{120\sin(x) + 168\cos(x) + 218}}$.



De optie zero (TI) of ROOT (Casio) geeft $x \approx 0{,}620$.

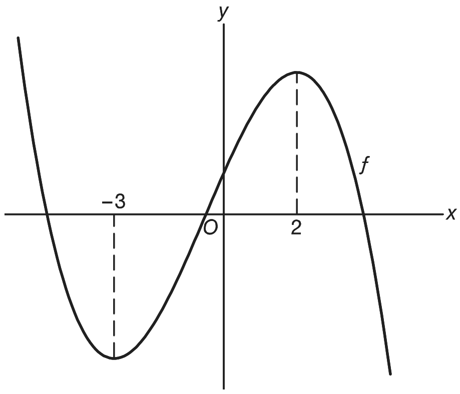


De lengte L is maximaal 20,6 dm bij $x \approx 0{,}620 \times \frac{180}{\pi} \approx 36°$.

12.4 De tweede afgeleide

bladzijde 128

39 **a**



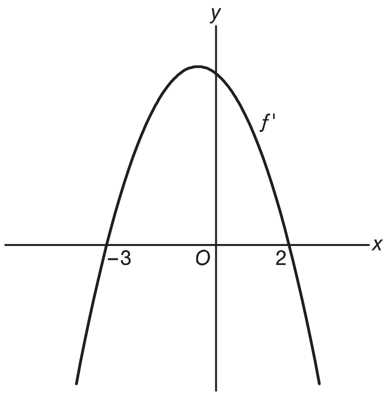
b De optie maximum geeft $x = 2$ en $y = 10\frac{1}{3}$.

De optie minimum geeft $x = -3$ en $y = -10\frac{1}{2}$.
dalend op $\langle \leftarrow, -3 \rangle$ en $\langle 2, \rightarrow \rangle$
stijgend op $\langle -3, 2 \rangle$

toenemend stijgend op $\langle -3, -\frac{1}{2} \rangle$

afnemend stijgend op $\langle -\frac{1}{2}, 2 \rangle$

c $f'(x) = -x^2 - x + 6$



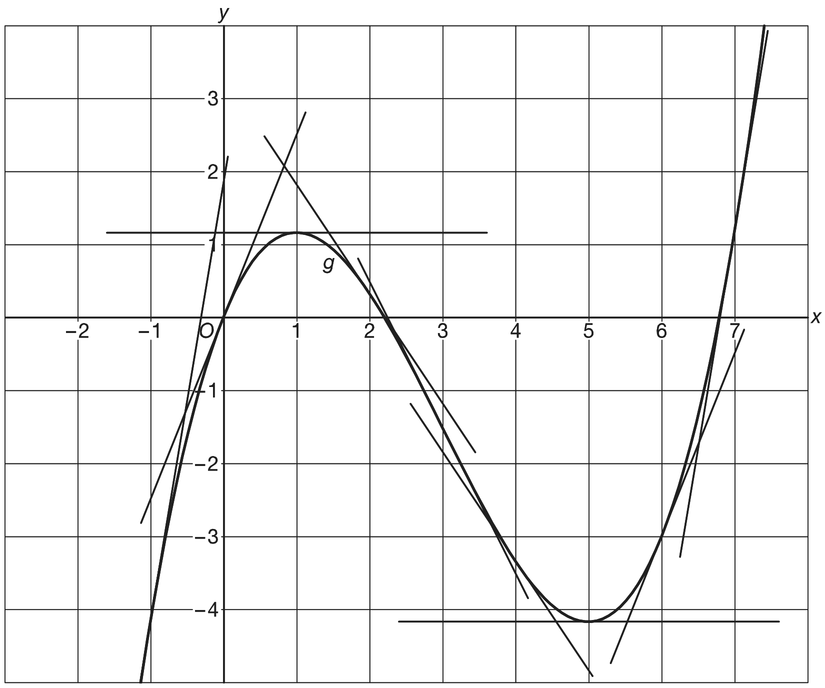
d $x_p = \dfrac{-3+2}{2} = -\frac{1}{2}$

e Bij $x = x_p$ gaat de grafiek van f over van toenemend stijgend in afnemend stijgend.

40 **a** $g(x) = \frac{1}{6}x^3 - 1\frac{1}{2}x^2 + 2\frac{1}{2}x$ geeft $g'(x) = \frac{1}{2}x^2 - 3x + 2\frac{1}{2}$

x	-1	0	1	2	3	4	5	6	7
$g(x)$	$-4\frac{1}{6}$	0	$1\frac{1}{6}$	$\frac{1}{3}$	$-1\frac{1}{2}$	$-3\frac{1}{3}$	$-4\frac{1}{6}$	-3	$1\frac{1}{6}$
$g'(x)$	6	$2\frac{1}{2}$	0	$-1\frac{1}{2}$	-2	$-1\frac{1}{2}$	0	$2\frac{1}{2}$	6

b



c De raaklijnen raken aan de bovenkant voor $x < 3$.
d De overgang vindt plaats in $(3, -1\frac{1}{2})$.
e onderkant
bovenkant

bladzijde 130

41 $\ln(e\sqrt{e}) = \ln(e^{1\frac{1}{2}}) = 1\frac{1}{2}$

42 $f'(e\sqrt{e}) = \dfrac{10 - 10 \cdot 1\frac{1}{2}}{(e\sqrt{e})^2} = \dfrac{10-15}{e^3} = \dfrac{-5}{e^3}$

$$\left.\begin{array}{l} y = -\dfrac{5}{e^3}x + b \\ \text{buigpunt}\left(e\sqrt{e}, \dfrac{15}{e\sqrt{e}}\right) \end{array}\right\} \dfrac{15}{e\sqrt{e}} = -\dfrac{5}{e^3} \cdot e\sqrt{e} + b$$

$$\dfrac{15}{e\sqrt{e}} + \dfrac{5}{e\sqrt{e}} = b$$

$$\dfrac{20}{e\sqrt{e}} = b$$

Dus buigraaklijn: $y = -\dfrac{5}{e^3}x + \dfrac{20}{e\sqrt{e}}$.

bladzijde 131

43 $f(x) = x\,e^x$ geeft $f'(x) = 1 \cdot e^x + x\,e^x = (1+x)e^x$
$f''(x) = 1 \cdot e^x + (1+x)e^x = (2+x)e^x$
$f''(x) = 0$ geeft $(2+x)e^x = 0$

$$2 + x = 0$$
$$x = -2$$

$f(-2) = -2e^{-2} = -\dfrac{2}{e^2}$

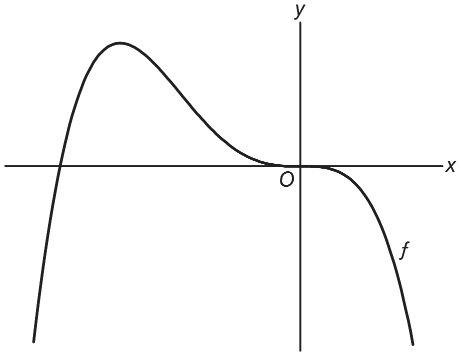
$f'(-2) = -e^{-2} = -\dfrac{1}{e^2}$

$$\left.\begin{array}{l} \text{Stel } k\text{: } y = -\dfrac{1}{e^2}x + b \\ \text{buigpunt}\left(-2, -\dfrac{2}{e^2}\right) \end{array}\right\} -\dfrac{2}{e^2} = -\dfrac{1}{e^2} \cdot -2 + b$$

$$-\dfrac{4}{e^2} = b$$

Dus de buigraaklijn is k: $y = -\dfrac{1}{e^2}x - \dfrac{4}{e^2}$.

44 **a**



b $f(x) = -\frac{1}{12}x^4 - \frac{1}{3}x^3$ geeft $f'(x) = -\frac{1}{3}x^3 - x^2$

$f''(x) = -x^2 - 2x$

$f''(x) = 0$ geeft $-x^2 - 2x = 0$

$x(-x - 2) = 0$

$x = 0 \ \vee \ x = -2$

$f(0) = 0$ en $f(-2) = 1\frac{1}{3}$

De buigpunten zijn $(0, 0)$ en $(-2, 1\frac{1}{3})$.

c $f'(0) = -\frac{1}{3} \cdot 0^3 - 0^2 = 0$ dus de buigraaklijn in $(0, 0)$ is horizontaal.

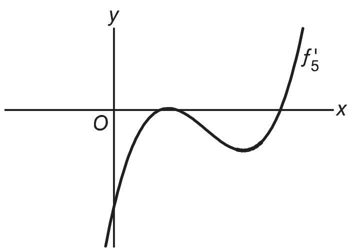
45 **a** $f_5(x) = \frac{1}{4}x^4 - 2x^3 + 5x^2 - 5x - 5$

$f_5'(x) = x^3 - 6x^2 + 10x - 5$

$f_5''(x) = 3x^2 - 12x + 10$

$f_5''(x) = 0$ geeft $3x^2 - 12x + 10 = 0$

$D = (-12)^2 - 4 \cdot 3 \cdot 10 = 144 - 120 = 24 > 0$



De grafiek van f_5 heeft twee buigpunten omdat f_5' twee extremen heeft.

b $f_6(x) = \frac{1}{4}x^4 - 2x^3 + 6x^2 - 5x - 5$

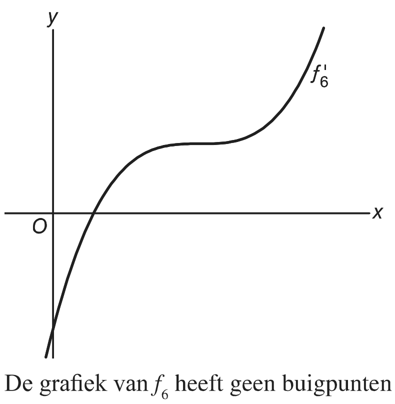
$f_6'(x) = x^3 - 6x^2 + 12x - 5$

$f_6''(x) = 3x^2 - 12x + 12$

$f_6''(x) = 0$ geeft $3x^2 - 12x + 12 = 0$

$D = (-12)^2 - 4 \cdot 3 \cdot 12 = 144 - 144 = 0$

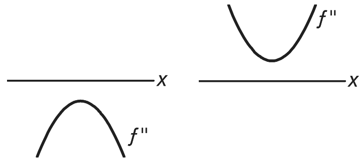
De vergelijking $f_6''(x) = 0$ heeft één oplossing.



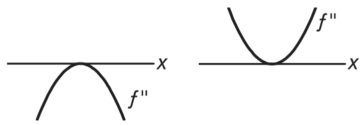
De grafiek van f_6 heeft geen buigpunten omdat f_6' geen extremen heeft.

c $f(x) = ax^4 + bx^3 + cx^2 + dx + e$
$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$
$f''(x) = 12ax^2 + 6bx + 2c$

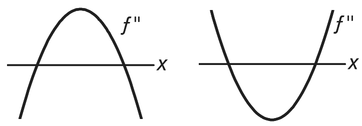
f' heeft geen extremen, dus de grafiek van f heeft geen buigpunten.



f' heeft geen extremen, dus de grafiek van f heeft geen buigpunten.



f' heeft twee extremen, dus de grafiek van f heeft twee buigpunten.



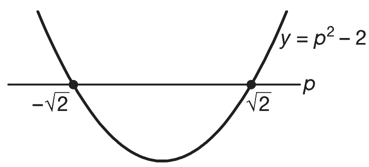
De grafiek van f heeft dus óf twee óf geen buigpunten.

46 $f_p(x) = x^4 + px^3 + \frac{3}{4}x^2 + 10$

$f_p'(x) = 4x^3 + 3px^2 + 1\frac{1}{2}x$

$f_p''(x) = 12x^2 + 6px + 1\frac{1}{2}$

$D = (6p)^2 - 4 \cdot 12 \cdot 1\frac{1}{2} = 36p^2 - 72$
$D \leq 0$ geeft $36p^2 - 72 \leq 0$
$p^2 - 2 \leq 0$
De grafiek van f_p heeft geen buigpunten als $-\sqrt{2} \leq p \leq \sqrt{2}$.



47 **a** $f(x) = \dfrac{5 + 10\ln(x)}{x}$ geeft

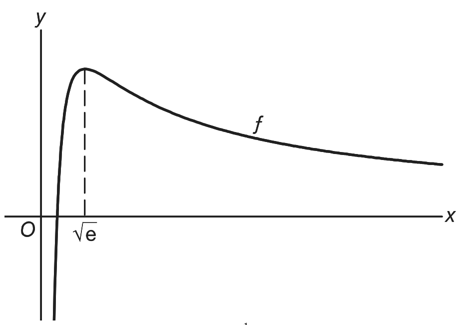
$$f'(x) = \frac{x \cdot \frac{10}{x} - (5 + 10\ln(x)) \cdot 1}{x^2} = \frac{10 - 5 - 10\ln(x)}{x^2} = \frac{5 - 10\ln(x)}{x^2}$$

$f'(x) = 0$ geeft $\dfrac{5 - 10\ln(x)}{x^2} = 0$

$5 - 10\ln(x) = 0$

$\ln(x) = \frac{1}{2}$

$x = e^{\frac{1}{2}} = \sqrt{e}$



max. is $f(\sqrt{e}) = \dfrac{5 + 10 \cdot \frac{1}{2}}{\sqrt{e}} = \dfrac{10}{\sqrt{e}}$

b $f'(x) = \dfrac{5 - 10\ln(x)}{x^2}$ geeft

$$f''(x) = \dfrac{x^2 \cdot \dfrac{-10}{x} - (5 - 10\ln(x)) \cdot 2x}{x^4} = \dfrac{-10x - 10x + 20x\ln(x)}{x^4}$$

$$= \dfrac{20x\ln(x) - 20x}{x^4} = \dfrac{20\ln(x) - 20}{x^3}$$

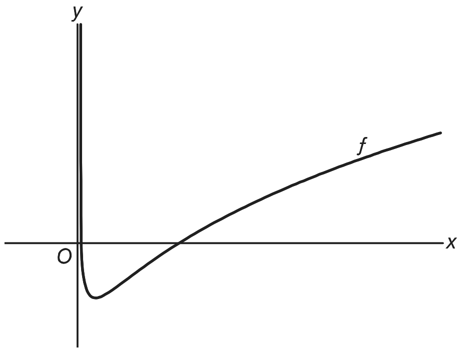
$f''(x) = 0$ geeft $\dfrac{20\ln(x) - 20}{x^3} = 0$

$20 \ln(x) - 20 = 0$
$\ln(x) = 1$
$x = \mathrm{e}$

$f(\mathrm{e}) = \dfrac{5 + 10\ln(\mathrm{e})}{\mathrm{e}} = \dfrac{15}{\mathrm{e}}$

Het buigpunt is $\left(\mathrm{e}, \dfrac{15}{\mathrm{e}}\right)$.

48 **a**



b $f(x) = (\ln(x))^2 + 2\ln(x) - 2$ geeft $f'(x) = 2\ln(x) \cdot \dfrac{1}{x} + 2 \cdot \dfrac{1}{x} = \dfrac{2}{x}(1 + \ln(x))$

$f'(x) = 0$ geeft $\dfrac{2}{x} \cdot (1 + \ln(x)) = 0$

$\ln(x) = -1$

$x = \mathrm{e}^{-1} = \dfrac{1}{\mathrm{e}}$

$f\left(\dfrac{1}{\mathrm{e}}\right) = 1 - 2 - 2 = -3$

De top is het punt $\left(\dfrac{1}{\mathrm{e}}, -3\right)$.

c $f''(x) = \dfrac{-2}{x^2}(1 + \ln(x)) + \dfrac{2}{x} \cdot \dfrac{1}{x} = \dfrac{-2}{x^2} - \dfrac{2}{x^2}\ln(x) + \dfrac{2}{x^2} = \dfrac{-2}{x^2} \cdot \ln(x)$

$f''(x)$ geeft $\dfrac{-2}{x} \cdot \ln(x) = 0$

$\ln(x) = 0$
$x = 1$

$\mathrm{rc}_{\text{raaklijn}} = f'(1) = 2$

$$\left.\begin{array}{l} \text{Stel } y = 2x + b \\ f(1) = -2 \text{ dus buigpunt } (1, -2) \end{array}\right\} \begin{array}{l} -2 = 2 + b \\ -4 = b \end{array}$$

De buigraaklijn is $y = 2x - 4$.

49 **a** $f(x) = 6x\,e^{-\frac{1}{24}x^3}$ geeft

$f'(x) = 6\,e^{-\frac{1}{24}x^3} + 6x\,e^{-\frac{1}{24}x^3} \cdot -\frac{1}{8}x^2 = (6 - \frac{3}{4}x^3)e^{-\frac{1}{24}x^3}$

$f'(x) = 0$ geeft $(6 - \frac{3}{4}x^3)e^{-\frac{1}{24}x^3} = 0$

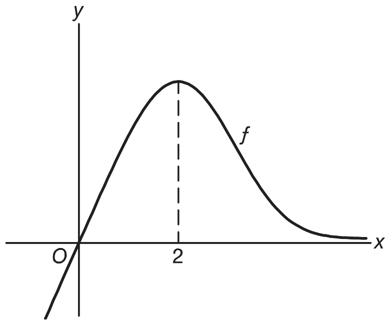
$6 - \frac{3}{4}x^3 = 0$

$\frac{3}{4}x^3 = 6$

$x^3 = 8$

$x = 2$

De x-coördinaat van de top is 2.



b $f''(x) = -\frac{9}{4}x^2\,e^{-\frac{1}{24}x^3} + (6 - \frac{3}{4}x^3) \cdot e^{-\frac{1}{24}x^3} \cdot -\frac{1}{8}x^2$

$= -\frac{9}{4}x^2\,e^{-\frac{1}{24}x^3} + (-\frac{3}{4}x^2 + \frac{3}{32}x^5)e^{-\frac{1}{24}x^3}$

$= (-3x^2 + \frac{3}{32}x^5)e^{-\frac{1}{24}x^3}$

$f''(x) = 0$ geeft $-3x^2 + \frac{3}{32}x^5 = 0$

$3x^2(\frac{1}{32}x^3 - 1) = 0$

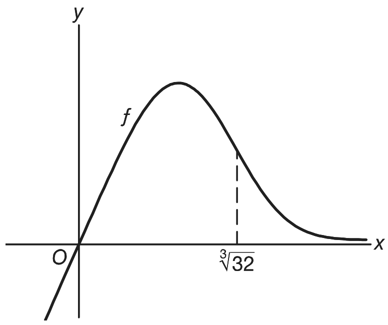
$3x^2 = 0 \;\vee\; \frac{1}{32}x^3 - 1 = 0$

$x = 0 \;\vee\; x^3 = 32$

$x = 0 \;\vee\; x = \sqrt[3]{32}$

v.n.

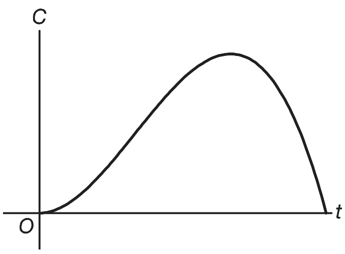
De x-coördinaat van het buigpunt is $\sqrt[3]{32}$.



12.5 Toepassingen van de tweede afgeleide

bladzijde 133

50 **a**



b Voer in $y_1 = -0{,}0004x^3 + 0{,}04x^2 + 0{,}28x$.
De optie maximum geeft $x = 70$ en $y = 78{,}4$.
C is maximaal 78 mg/l na 70 minuten.

c $\dfrac{dC}{dt} = -0{,}0012t^2 + 0{,}08t + 0{,}28$

$\dfrac{dC}{dt}$ is de snelheid in mg/liter/minuut waarmee C verandert.

Voer in $y_1 = -0{,}0012x^2 + 0{,}08x + 0{,}28$.
De optie maximum geeft $x \approx 33{,}33$ en $y \approx 1{,}61$.

$\dfrac{dC}{dt}$ is maximaal na ongeveer 33 minuen.

bladzijde 134

51 **a** $\dfrac{dN}{dt} = 2e^{-0{,}02t} > 0$ omdat $2 > 0 \;\wedge\; e^{-0{,}02t} > 0$.

$\dfrac{d}{dt}\left(\dfrac{dN}{dt}\right) = -0{,}04e^{-0{,}02t} < 0$ omdat $-0{,}04 < 0 \;\wedge\; e^{-0{,}02t} > 0$.

b Uit $\frac{dN}{dt} > 0$ volgt dat N een stijgende functie is.

Uit $\frac{d}{dt}\left(\frac{dN}{dt}\right) < 0$ volgt dat $\frac{dN}{dt}$ een dalende functie is.

$$\left.\begin{array}{l} N \text{ stijgt} \\ \frac{dN}{dt} \text{ daalt} \end{array}\right\} N \text{ is afnemend stijgend.}$$

c $\frac{dN}{dt} > 0$ en $\frac{d}{dt}\left(\frac{dN}{dt}\right) > 0$

d
$$\left.\begin{array}{l} \frac{dN}{dt} < 0 \\ \frac{d}{dt}\left(\frac{dN}{dt}\right) > 0 \end{array}\right\} N \text{ is afnemend dalend}$$

$$\left.\begin{array}{l} \frac{dN}{dt} < 0 \\ \frac{d}{dt}\left(\frac{dN}{dt}\right) < 0 \end{array}\right\} N \text{ is toenemend dalend}$$

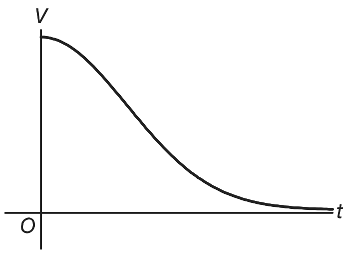
52 $T = 20 + 80\,e^{-0,2t}$ geeft $\frac{dT}{dt} = 80\,e^{-0,2t} \cdot -0,2 = -16\,e^{-0,2t}$

$\frac{d}{dt}\left(\frac{dT}{dt}\right) = -16\,e^{-0,2t} \cdot -0,2 = 3,2\,e^{-0,2t}$

$$\left.\begin{array}{l} \frac{dT}{dt} = -16\,e^{-0,2t} < 0 \\ \frac{d}{dt}\left(\frac{dT}{dt}\right) = 3,2\,e^{-0,2t} > 0 \end{array}\right\} T \text{ is afnemend dalend}$$

bladzijde 135

53 **a**



b Voer in $y_1 = 100\,e^{-0,01x^2}$ en $y_2 = 50$.
De optie intersect geeft $x \approx 8,33$.
8,33 minuten = 8,33 · 60 ≈ 500 seconden
Na ongeveer 500 seconden is de helft weggestroomd.

c $\frac{dV}{dt} = 100 \cdot e^{-0,01t^2} \cdot -0,02t = -2t\,e^{-0,01t^2}$

$\frac{d}{dt}\left(\frac{dV}{dt}\right) = -2e^{-0,01t^2} + -2t \cdot e^{-0,01t^2} \cdot -0,02t$
$= -2e^{-0,01t^2} + 0,04t^2e^{-0,01t^2} = (0,04t^2 - 2)e^{-0,01t^2}$

$\frac{d}{dt}\left(\frac{dV}{dt}\right) = 0$ geeft $(0,04t^2 - 2)e^{-0,01t^2} = 0$
$0,04t^2 - 2 = 0$
$t^2 = 50$
$t = \sqrt{50} \vee t = -\sqrt{50}$
voldoet niet

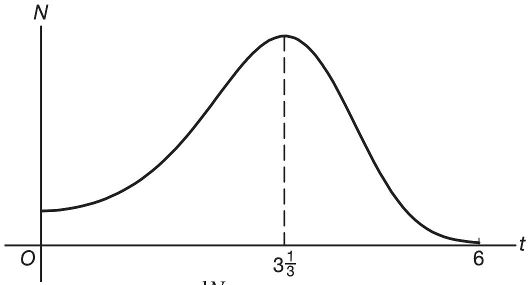
De uitstroomsnelheid is maximaal na $\sqrt{50}$ minuten, dus na ongeveer 424 seconden.

d $\left[\frac{dV}{dt}\right]_{t=\sqrt{50}} \approx -8,58$.
De maximale uitstroomsnelheid is dus 8,58 l/minuut.
De helft van de maximale snelheid is 4,29 l/minuut.

Voer in $y_1 = -2x\,\mathrm{e}^{-0{,}01x^2}$ en $y_2 = -4{,}29$.
De optie intersect geeft $x \approx 2{,}26$ en $x \approx 13{,}59$.

Ongeveer $13{,}59 - \sqrt{50} \approx 6{,}52$ minuten ≈ 391 seconden na $t = \sqrt{50}$ is de uitstroomsnelheid afgenomen tot de helft van de maximale snelheid.

54 **a**



$N = \mathrm{e}^{-0{,}1t^3+0{,}5t^2}$ geeft $\dfrac{\mathrm{d}N}{\mathrm{d}t} = \mathrm{e}^{-0{,}1t^3+0{,}5t^2} \cdot (-0{,}3t^2 + t) = (-0{,}3t^2 + t)\mathrm{e}^{-0{,}1t^3+0{,}5t^2}$

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}t} = 0 \text{ geeft } & (-0{,}3t^2 + t)\mathrm{e}^{-0{,}1t^3+0{,}5t^2} = 0\\ & -0{,}3t^2 + t = 0\\ & -t(0{,}3t - 1) = 0\\ & -t = 0 \ \vee\ 0{,}3t - 1 = 0\\ & t = 0 \ \vee\ t = 3\tfrac{1}{3}\end{aligned}$$

Na $3\frac{1}{3} \cdot 24 = 80$ uur is het aantal bacteriën maximaal.

b Voer in $y_1 = (-0{,}3x^2 + x)\mathrm{e}^{-0{,}1x^3+0{,}5x^2}$.
De optie maximum geeft $x \approx 2{,}41$ en $y \approx 3{,}00$.
De snelheid is maximaal na $2{,}41 \cdot 24 \approx 58$ uur.
De maximale snelheid is ongeveer 3 miljoen/dag ofwel 125 000 bacteriën per uur.

c $\left[\dfrac{\mathrm{d}N}{\mathrm{d}t}\right]_{t=\frac{100}{24}} \approx -4{,}43$ miljoen/dag.

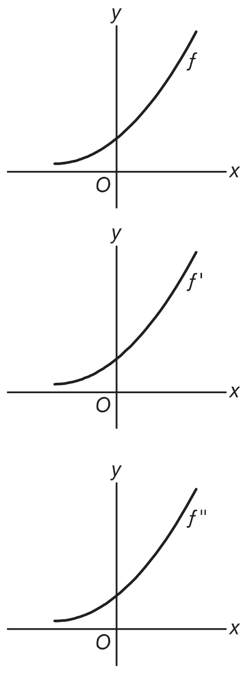
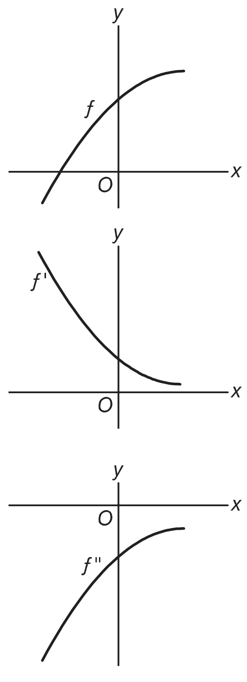
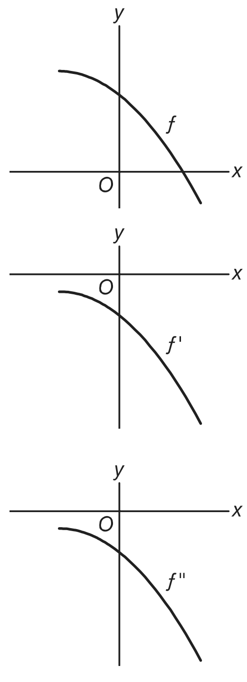
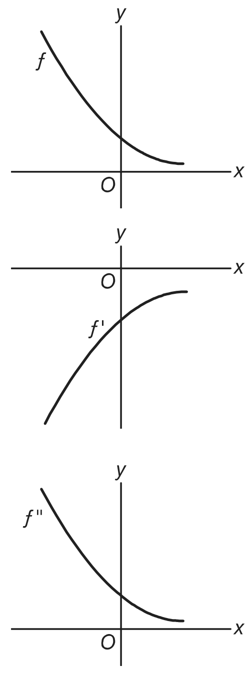
Het aantal bacteriën neemt af met ongeveer $\dfrac{4{,}43 \cdot 10^6}{24 \cdot 60} \approx 3074$ bacteriën/minuut.

d $\dfrac{\mathrm{d}N}{\mathrm{d}t} = (-0{,}3t^2 + t)\mathrm{e}^{-0{,}1t^3+0{,}5t^2}$ geeft

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right) &= (-0{,}6t + 1) \cdot \mathrm{e}^{-0{,}1t^3+0{,}5t^2} + (-0{,}3t^2 + t)^2 \cdot \mathrm{e}^{-0{,}1t^3+0{,}5t^2} \cdot (-0{,}3t^2 + t)\\ &= (-0{,}6t + 1)\mathrm{e}^{-0{,}1t^3+0{,}5t^2} + (-0{,}3t^2 + t)^2\mathrm{e}^{-0{,}1t^3+0{,}5t^2}\\ &= (-0{,}6t + 1 + (-0{,}3t^2 + t)^2)\mathrm{e}^{-0{,}1t^3+0{,}5t^2}\end{aligned}$$

$$\left.\begin{aligned}\left[\frac{\mathrm{d}N}{\mathrm{d}t}\right]_{t=\frac{110}{24}} &\approx -4{,}12 < 0\\ \left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)\right]_{t=\frac{110}{24}} &\approx 2{,}89 > 0\end{aligned}\right\}\ N \text{ is afnemend dalend na 110 uur.}$$

55

$f''(x) > 0$ voor elke x. $f'(x) > 0$ voor elke x. De grafiek van f is toenemend stijgend.	$f''(x) < 0$ voor elke x. $f'(x) > 0$ voor elke x. De grafiek van f is afnemend stijgend.	$f''(x) < 0$ voor elke x. $f'(x) < 0$ voor elke x. De grafiek van f is toenemend dalend.	$f''(x) > 0$ voor elke x. $f'(x) < 0$ voor elke x. De grafiek van f is afnemend dalend.
			
Bijvoorbeeld: $f(x) = \mathrm{e}^x$ $f'(x) = \mathrm{e}^x$ $f''(x) = \mathrm{e}^x$	Bijvoorbeeld: $f(x) = -\mathrm{e}^{-x} + 3$ $f'(x) = \mathrm{e}^{-x}$ $f''(x) = -\mathrm{e}^{-x}$	Bijvoorbeeld: $f(x) = -\mathrm{e}^x + 3$ $f'(x) = -\mathrm{e}^x$ $f''(x) = -\mathrm{e}^x$	Bijvoorbeeld: $f(x) = \mathrm{e}^{-x}$ $f'(x) = -\mathrm{e}^{-x}$ $f''(x) = \mathrm{e}^{-x}$

56 **a** Bijvoorbeeld $f(x) = x^3 + 2x^2 - 5$.

b $\left.\begin{array}{l} f(x) = x^3 + 2x^2 + c \\ \text{door } (0, -2) \end{array}\right\} -2 = 0 + 0 + c$

$-2 = c$

Dus $f(x) = x^3 + 2x^2 - 2$.

bladzijde 136

57 **a** $\left.\begin{array}{l} v = \dfrac{\mathrm{d}s}{\mathrm{d}t} \\ s = 0{,}2t^2 + 0{,}1t \text{ geeft } \dfrac{\mathrm{d}s}{\mathrm{d}t} = 0{,}4t + 0{,}1 \end{array}\right\} v = 0{,}4t + 0{,}1$

b $v(4) = 1{,}7$ m/s

$v(5) = 2{,}1$ m/s

c De snelheid is met $2{,}1 - 1{,}7 = 0{,}4$ m/s toegenomen op het interval $[4, 5]$.

$v(6) = 2{,}5$ m/s

Op $[5, 6]$ is de snelheid toegenomen met $2{,}5 - 2{,}1 = 0{,}4$ m/s.

d $v(t + 1) - v(t) = 0{,}4(t + 1) + 0{,}1 - (0{,}4t + 0{,}1) = 0{,}4t + 0{,}4 + 0{,}1 - 0{,}4t - 0{,}1 = 0{,}4$.

Op elk interval $[t, t + 1]$ is de toename 0,4 m/s.

bladzijde 137

58 Stel $a(t) = mt + n$.

$a(0) = 0$ en $a(20) = 5$, dus $a(t) = \frac{1}{4}t$

$\left.\begin{array}{l} a(t) = \frac{1}{4}t \\ v(0) = 1 \end{array}\right\} v(t) = \frac{1}{2}t^2 + 1$

en dit geeft $s(t) = \frac{1}{6}t^3 + t$.

$s(20) = \frac{1}{6} \cdot 20^3 + 20 = 1353\frac{1}{3}$

Dus gedurende de eerste 20 seconden wordt ongeveer 1350 m afgelegd.

59 **a** $\left.\begin{array}{l} a = -t^2 + 6t \\ v(0) = 0 \end{array}\right\} v(t) = \frac{1}{3}t^3 + 3t^2$

$v(6) = -\frac{1}{3} \cdot 6^3 + 3 \cdot 6^2 = 36$ m/s

b $\left.\begin{array}{l} v(t) = -\frac{1}{3}t^3 + 3t^2 \\ s(0) = 0 \end{array}\right\} s(t) = -\frac{1}{12}t^4 + t^3$

$s(6) = -\frac{1}{12} \cdot 6^4 + 6^3 = 108$ m

c $v(6) = 36$ m/s

$s(10) = s(6) + 4 \cdot v(6) = 108 + 4 \cdot 36 = 252$ m

d Voor $t \geq 6$ geldt $s(t) = s(6) + (t - 6) \cdot v(6)$.

$s(t) = 500$ geeft $108 + (t - 6) \cdot 36 = 500$

$36t - 108 = 500$

$36t = 608$

$t = 16\frac{8}{9}$

Op $t = 16\frac{8}{9}$ is 500 m afgelegd.

bladzijde 138

60 **a** $\left.\begin{array}{l} v(0) = 108 \text{ km/uur} = 30 \text{ m/s} \\ v(6) = 0 \end{array}\right\} a(t) = \dfrac{0-30}{6-0} = -5 \text{ m/s}^2$

$\left.\begin{array}{l} a(t) = -5 \\ v(0) = 30 \end{array}\right\} v(t) = -5t + 30$ m/s

$\left.\begin{array}{l} v(t) = -5t + 30 \\ s(0) = 0 \end{array}\right\} s(t) = -2{,}5t^2 + 30t$ m

$s(6) = 90$ m

De remweg is 90 m.

b Stel $a(t) = a$ m/s^2

$\left.\begin{array}{l} a(t) = a \\ v(0) = 30 \end{array}\right\} v(t) = at + 30$

$\left.\begin{array}{l} v(t) = at + 30 \\ s(0) = 0 \end{array}\right\} s(t) = \frac{1}{2}at^2 + 30t$

$v(t) = 0$ geeft $at + 30 = 0$

$t = \dfrac{-30}{a}$

$$s\left(\frac{-30}{a}\right) = \frac{1}{2}a\left(\frac{-30}{a}\right)^2 + 30 \cdot \frac{-30}{a} = \frac{450}{a} - \frac{900}{a} = \frac{-450}{a}$$

$s\left(\dfrac{-30}{a}\right) = 60$ geeft $\dfrac{-450}{a} = 60$

$a = \dfrac{-450}{60} = -7{,}5$ m/s^2

$t = \dfrac{-30}{a} = \dfrac{-30}{-7{,}5} = 4$

De remtijd is 4 seconden.

61 **a** Voor de auto geldt:

$\left.\begin{array}{l} a(t) = 1{,}5 \\ v_A(0) = 0 \end{array}\right\} v_A(t) = 1{,}5t$

$\left.\begin{array}{l} v_A(t) = 1{,}5t \\ s_A(0) = 0 \end{array}\right\} s_A(t) = 0{,}75t^2$

Voor de brommer geldt: $s_B(t) = 10t$.

$s_A(t) = s_B(t)$ geeft $0{,}75t^2 = 10t$

$0{,}75t^2 - 10t = 0$

$t(0{,}75t - 10) = 0$

$t = 0 \vee 0{,}75t = 10$

$t = 0 \vee t = 13\frac{1}{3}$

De auto heeft de brommer na $13\frac{1}{3}$ seconde ingehaald.

b $v_A(13\frac{1}{3}) = 1{,}5 \cdot 13\frac{1}{3} = 20$ m/s $= 72$ km/uur

62 **a** $v(t)$

b $\left.\begin{array}{l} a(t) = -4 \\ v(0) = 25 \end{array}\right\} v(t) = -4t + 25$

$v(t) = 0$ geeft $-4t + 25 = 0$

$-4t = -25$

$t = 6\frac{1}{4}$

Dus $A = (6\frac{1}{4}, 0)$.

$v(0) = 25$ dus $B = (0, 25)$.

c opp($\triangle OAB$) $= \frac{1}{2} \cdot 25 \cdot 6\frac{1}{4} = 78\frac{1}{8}$

opp($\triangle OAB$) is de afgelegde afstand.

12.6 Diagnostische toets

bladzijde 140

1 **a** $f(x) = \dfrac{3x-7}{x^2+2}$ geeft

$$f'(x) = \frac{(x^2+2)\cdot 3 - (3x-7)\cdot 2x}{(x^2+2)^2} = \frac{3x^2+6-6x^2+14x}{(x^2+2)^2} = \frac{-3x^2+14x+6}{(x^2+2)^2}$$

b $g(x) = 3x - \dfrac{2}{x+4}$ geeft $g'(x) = 3 - \dfrac{(x+4)\cdot 0 - 2\cdot 1}{(x+4)^2} = 3 + \dfrac{2}{(x+4)^2}$

c $h(x) = \dfrac{3x^2}{4x-2}$ geeft $h'(x) = \dfrac{(4x-2)\cdot 6x - 3x^2\cdot 4}{(4x-2)^2} = \dfrac{24x^2-12x-12x^2}{(4x-2)^2} = \dfrac{12x^2-12x}{(4x-2)^2}$

2 **a** $f(x) = \dfrac{x^2-3x+18}{2x-4}$ geeft

$$f'(x) = \frac{(2x-4)(2x-3)-(x^2-3x+18)\cdot 2}{(2x-4)^2} = \frac{4x^2-6x-8x+12-2x^2+6x-36}{(2x-4)^2} = \frac{2x^2-8x-24}{(2x-4)^2}$$

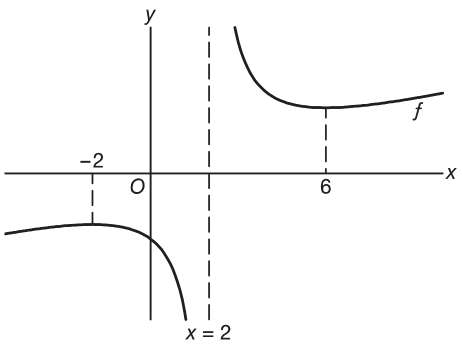
$f'(x) = 0$ geeft $\dfrac{2x^2-8x-24}{(2x-4)^2} = 0$

$2x^2 - 8x - 24 = 0$

$x^2 - 4x - 12 = 0$

$(x+2)(x-6) = 0$

$x = -2 \lor x = 6$



max. is $f(-2) = -3\frac{1}{2}$

min. is $f(6) = 4\frac{1}{2}$

$B_f = \langle\leftarrow, -3\frac{1}{2}] \cup [4\frac{1}{2}, \rightarrow\rangle$, dat wil zeggen $y \leq -3\frac{1}{2} \lor y \geq 4\frac{1}{2}$.

b $\left.\begin{array}{l} f'(0) = \dfrac{-24}{16} = -1\frac{1}{2} \\ f(0) = \dfrac{18}{-4} = -4\frac{1}{2} \end{array}\right\}$ raaklijn: $y = -1\frac{1}{2}x - 4\frac{1}{2}$

3 **a** $N = \dfrac{1600t}{t^2+100} + 10$ geeft

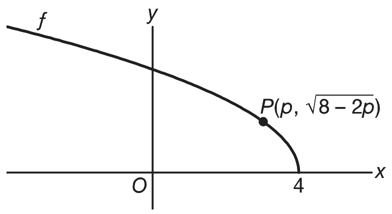
$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{(t^2+100)\cdot 1600 - 1600t\cdot 2t}{(t^2+100)^2} = \frac{1600t^2 + 160000 - 3200t^2}{(t^2+100)^2} = \frac{-1600t^2+160000}{(t^2-100)^2}$$

$\left[\dfrac{\mathrm{d}N}{\mathrm{d}t}\right]_{t=5} = 7{,}68 > 0$, dus N neemt toe op $t = 5$.

b $\left[\dfrac{\mathrm{d}N}{\mathrm{d}t}\right]_{t=15} = \dfrac{-1600\cdot 225 + 160000}{(225+100)^2} = \dfrac{-200000}{105625} \approx -1{,}9$

Dus N neemt af met een snelheid van ongeveer 1,9 per dag.

4 **a**

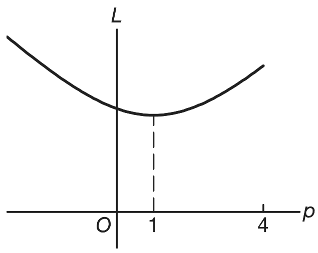


$L = OP = \sqrt{p^2 + (\sqrt{8-2p})^2} = \sqrt{p^2+8-2p} = \sqrt{p^2-2p+8}$

b $\dfrac{\mathrm{d}L}{\mathrm{d}p} = \dfrac{1}{2\sqrt{p^2-2p+8}}\cdot(2p-2) = \dfrac{p-1}{\sqrt{p^2-2p+8}}$

$\dfrac{\mathrm{d}L}{\mathrm{d}p} = 0$ geeft $p - 1 = 0$

$p = 1$



De minimale waarde van L is $L(1) = \sqrt{7}$.

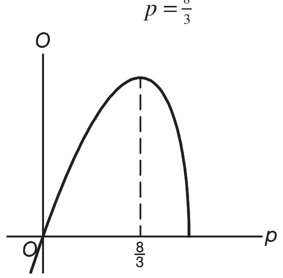
c $O = p\cdot\sqrt{8-2p}$ geeft $\dfrac{\mathrm{d}O}{\mathrm{d}p} = 1\cdot\sqrt{8-2p} + p\cdot\dfrac{1}{2\sqrt{8-2p}}\cdot -2 = \sqrt{8-2p} - \dfrac{p}{\sqrt{8-2p}}$

$\dfrac{\mathrm{d}O}{\mathrm{d}p} = 0$ geeft $\sqrt{8-2p} = \dfrac{p}{\sqrt{8-2p}}$

$8 - 2p = p$

$-3p = -8$

$p = \frac{8}{3}$



De maximale waarde van O is $O(\frac{8}{3}) = \frac{8}{3}\sqrt{\frac{8}{3}}$.

5 $L = CD = f(p) - g(p) = \sqrt{8-2p^2} - (\frac{1}{2}p + 2) = \sqrt{8-2p^2} - \frac{1}{2}p - 2$

$$\frac{dL}{dp} = \frac{1}{2\sqrt{8-2p^2}} \cdot -4p - \frac{1}{2} = \frac{-2p}{\sqrt{8-2p^2}} - \frac{1}{2}$$

$\frac{dL}{dp} = 0$ geeft $\frac{-2p}{\sqrt{8-2p^2}} - \frac{1}{2} = 0$

$$\frac{-2p}{\sqrt{8-2p^2}} = \frac{1}{2}$$

$$-4p = \sqrt{8-2p^2}$$

kwadrateren geeft

$16p^2 = 8 - 2p^2$

$18p^2 = 8$

$p^2 = \frac{4}{9}$

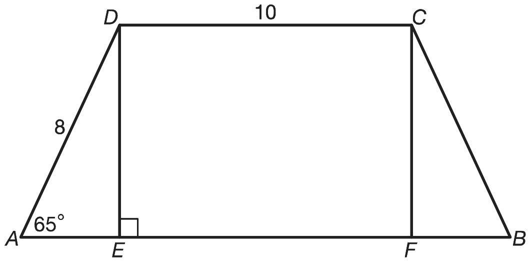
$p = -\frac{2}{3} \vee p = \frac{2}{3}$

vold. niet

Uit de figuur volgt dat de lengte van CD maximaal is voor $p = -\frac{2}{3}$.

bladzijde 141

6 **a**



$\sin 65° = \frac{DE}{8}$, dus $DE = 8 \sin 65°$

$\cos 65° = \frac{AE}{8}$, dus $AE = 8 \cos 65°$

$O(ABCD) = 2 \cdot O(\triangle AED) + O(EFCD)$

$= 2 \cdot \frac{1}{2} \cdot 8 \cos 65° \cdot 8 \sin 65° + 10 \cdot 8 \sin 65°$

$= 64 \cos 65° \sin 65° + 80 \sin 65° \approx 97{,}0$

b $\sin(x) = \frac{DE}{8}$, dus $DE = 8 \sin(x)$

$\cos(x) = \frac{AE}{8}$, dus $AE = 8 \cos(x)$

$O(ABCD) = 2 \cdot O(\triangle AED) + O(EFCD)$

$= 2 \cdot \frac{1}{2} \cdot 8 \cos(x) \cdot 8 \sin(x) + 10 \cdot 8 \sin(x)$

$= 64 \cos(x) \sin(x) + 80 \sin(x)$

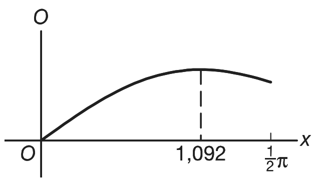
Dus $O = 80 \sin(x) + 64 \sin(x) \cos(x)$.

c $\frac{dO}{dx} = 80 \cos(x) + 64 \cos(x) \cdot \cos(x) + 64 \sin(x) \cdot -\sin(x)$

$= 80 \cos(x) + 64 \cos^2(x) - 64 \sin^2(x)$

Voer in $y_1 = 80 \cos(x) + 64 \cos^2(x) - 64 \sin^2(x)$.

De optie zero (TI) of ROOT (Casio) geeft $x \approx 1{,}092$.



$O(1{,}092) \approx 97{,}174$

De maximale oppervlakte is $O(1{,}092) \approx 97{,}17$.

7 $f(x) = x^4 - x^3 - 9x^2 - 5x$

$f'(x) = 4x^3 - 3x^2 - 18x - 5$

$f''(x) = 12x^2 - 6x - 18$

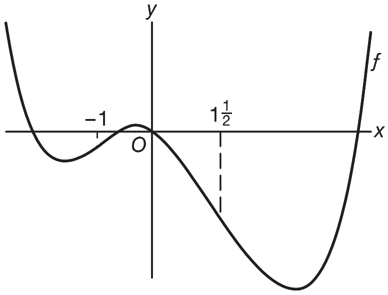
$f''(x) = 0$ geeft $12x^2 - 6x - 18 = 0$

$2x^2 - x - 3 = 0$

$D = 1 - 4 \cdot 2 \cdot -3 = 25$

$x = \dfrac{1-5}{4} \vee x = \dfrac{1+5}{4}$

$x = -1 \vee x = 1\frac{1}{2}$



$\left.\begin{array}{l} f'(-1) = 6 \text{ geeft raaklijn } y = 6x + b \\ f(-1) = -2 \text{ dus buigpunt } (-1, -2) \end{array}\right\} \begin{array}{l} -2 = -6 + b \\ 4 = b \end{array}$

Dus buigraaklijn: $y = 6x + 4$.

$\left.\begin{array}{l} f'(1\frac{1}{2}) = -25\frac{1}{4} \text{ geeft raaklijn } y = -25\frac{1}{4}x + b \\ f(1\frac{1}{2}) = -26\frac{1}{16} \text{ dus buigpunt } (1\frac{1}{2}, -26\frac{1}{16}) \end{array}\right\} \begin{array}{l} -26\frac{1}{16} = -25\frac{1}{4} \cdot 1\frac{1}{2} + b \\ 11\frac{13}{16} = b \end{array}$

Dus buigraaklijn: $y = -25\frac{1}{4}x + 11\frac{13}{16}$.

8 $f(x) = 4x\,e^{-\frac{1}{6}x^2}$

$f'(x) = 4\,e^{-\frac{1}{6}x^2} + 4x \cdot e^{-\frac{1}{6}x^2} \cdot -\frac{1}{3}x = 4\,e^{-\frac{1}{6}x^2} - \frac{4}{3}x^2 \cdot e^{-\frac{1}{6}x^2} = (4 - \frac{4}{3}x^2)e^{-\frac{1}{6}x^2}$

$f''(x) = -\frac{8}{3}x \cdot e^{-\frac{1}{6}x^2} + (4 - \frac{4}{3}x^2) \cdot e^{-\frac{1}{6}x^2} \cdot -\frac{1}{3}x = -\frac{8}{3}x \cdot e^{-\frac{1}{6}x^2} + (-\frac{4}{3}x + \frac{4}{9}x^3)e^{-\frac{1}{6}x^2}$

$= (-\frac{8}{3}x - \frac{4}{3}x + \frac{4}{9}x^3)e^{-\frac{1}{6}x^2} = (-4x + \frac{4}{9}x^3)e^{-\frac{1}{6}x^2}$

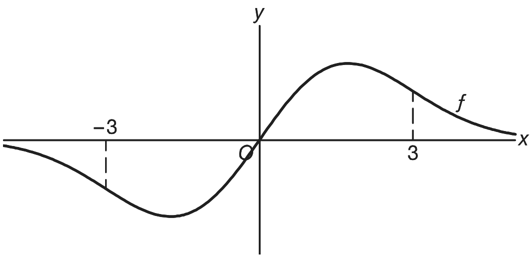
$f''(x) = 0$ geeft $(-4x + \frac{4}{9}x^3)e^{-\frac{1}{6}x^2} = 0$

$-4x + \frac{4}{9}x^3 = 0$

$\frac{4}{9}x(x^2 - 9) = 0$

$\frac{4}{9}x(x+3)(x-3) = 0$

$x = 0 \vee x = -3 \vee x = 3$



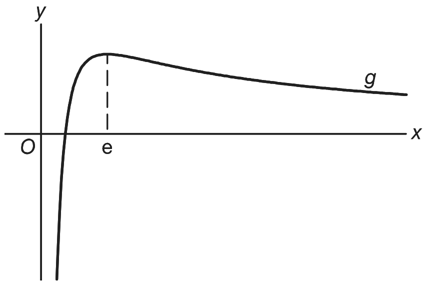
$f(-3) = -12e^{-1\frac{1}{2}}$ dus buigpunt $(-3, -12e^{-1\frac{1}{2}})$.

$f(0) = 0$ dus buigpunt $(0, 0)$.

$f(3) = 12e^{-1\frac{1}{2}}$ dus buigpunt $(3, 12e^{-1\frac{1}{2}})$.

9 $g(x)=\dfrac{\ln(x)}{x}$ geeft $g'(x)=\dfrac{x\cdot\frac{1}{x}-\ln(x)\cdot 1}{x^2}=\dfrac{1-\ln(x)}{x^2}$

$g'(x)=0$ geeft $1-\ln(x)=0$
$\ln(x)=1$
$x=\mathrm{e}$



$g(\mathrm{e})=\dfrac{1}{\mathrm{e}}$, dus top $\left(\mathrm{e},\dfrac{1}{\mathrm{e}}\right)$.

$g'(x)=\dfrac{1-\ln(x)}{x^2}$ geeft

$$g''(x)=\dfrac{x^2\cdot-\frac{1}{x}-(1-\ln(x))\cdot 2x}{(x^2)^2}=\dfrac{-x-2x+2x\ln(x)}{x^4}=\dfrac{-3x+2x\ln(x)}{x^4}=\dfrac{-3+2\ln(x)}{x^3}$$

$g''(x)=0$ geeft $-3+2\ln(x)=0$

$\ln(x)=1\frac{1}{2}$

$x=\mathrm{e}^{1\frac{1}{2}}=\mathrm{e}\sqrt{\mathrm{e}}$

$g(\mathrm{e}\sqrt{\mathrm{e}})=\dfrac{1\frac{1}{2}}{\mathrm{e}\sqrt{\mathrm{e}}}=\dfrac{3}{2\mathrm{e}\sqrt{\mathrm{e}}}$, dus buigpunt $\left(\mathrm{e}\sqrt{\mathrm{e}},\dfrac{3}{2\mathrm{e}\sqrt{\mathrm{e}}}\right)$.

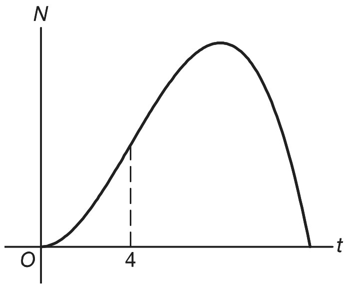
10 $N=480t^2-40t^3$

$\dfrac{\mathrm{d}N}{\mathrm{d}t}=960t-120t^2$

$\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}N}{\mathrm{d}t}\right)=960-240t$

$\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}N}{\mathrm{d}t}\right)=0$ geeft $960-240t=0$

$240t=960$
$t=4$



De snelheid waarmee N toeneemt is maximaal voor $t=4$, dus om 13.00 uur.

11 $\left.\begin{array}{l}a(t)=-9{,}8 \text{ geeft } v(t)=-9{,}8t+v(0)\\ v(0)=10\end{array}\right\}\ v(t)=-9{,}8t+10$

$v(t)=-9{,}8t+10$ geeft $s(t)=-4{,}9t^2+10t$
$s(t)=-150$ geeft $-4{,}9t^2+10t=-150$
$4{,}9t^2-10t-150=0$
$D=100-4\cdot 4{,}9\cdot -150=3040$

$t=\dfrac{10-\sqrt{3040}}{9{,}8}\ \vee\ t=\dfrac{10+\sqrt{3040}}{9{,}8}$

$t\approx -4{,}61\ \vee\ t\approx 6{,}65$
voldoet niet
Na 6,65 seconden valt de steen op de bodem van de kloof.

12 **a** $a(t) = 0{,}2t + 1$ geeft $v(t) = 0{,}1t^2 + t + v(0)$
$v(0) = 0$ $\Big\}\ v(t) = 0{,}1t^2 + t$
$t = 5$ geeft $v(5) = 0{,}1 \cdot 25 + 5 = 7{,}5$
Dus de snelheid op $t = 5$ is 7,5 m/s.

b $v(t) = 0{,}1t^2 + t$ geeft $s(t) = \frac{1}{30}t^3 + \frac{1}{2}t^2$
$s(12) = \frac{1}{30} \cdot 12^3 + \frac{1}{2} \cdot 12^2 = 129{,}6$
Dus de afgelegde weg is 129,6 m.